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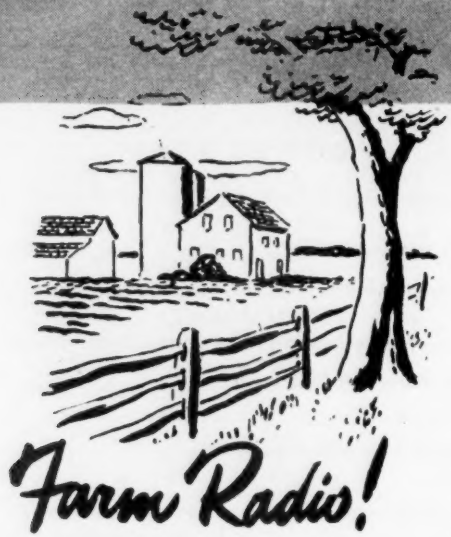

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Vol. 239 MARCH, 1945 No. 3

CERTAIN APPLICATIONS OF PHYSICAL PRINCIPLES TO THE PLAYING OF MUSICAL INSTRUMENTS.*

BY

W. F. G. SWANN,

Director, Bartol Research Foundation of The Franklin Institute, Swarthmore, Pennsylvania.

(*Continued from February issue.*)

THE PIANO.

I now intend to make a few remarks concerning the much debated question of the piano and its tone. I wish to concentrate on one point only, namely the question of what is involved in that old controversy as to whether or not the player has it within his power to modify the quality of tone produced in the case of single notes played with the same intensity. As an example of one extreme in schools of thought, I may quote from Sir James Jeans, who, summing up the situation from the scientific standpoint, remarks: "It seems clear that, so long as he confines himself to striking single tones, the greatest virtuoso has no greater range of effects at his disposal than the child strumming at its five finger exercises." On the other hand, quoting from a recent book ("The Singing Touch," Betak Reeder, Galaxy Music Corporation, New York) written primarily from the standpoint of the artist, we read: "Unquestionably the force with which the piano key is struck influences both the quality and the loudness of the tone. But to say that force is the sole factor in the manipulation of the key is a fallacy. Because, over and above force, there is that something we call *touch*, which alone is capable of bringing out the best qualities of tone."

Unfortunately, the artist, when using the phraseology of science,

* Presented at the Stated Meeting held on March 15, 1944. The text of this paper will be presented in semi-popular vein. However, the paper will be supplemented by more technical footnotes and by an appendix, which will appear at the end of the third section of the paper, and to which reference will be made in the text.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

is apt to use scientific terms to mean things entirely different from those meant by the scientist himself when using those terms. One reads of such things as *volume*, said to be distinguished from *loudness* by the "swelling out of the sound at the beginning and the end." I could quote from various sources all sorts of curious statements of this kind, but I hesitate to poke fun in individual cases at those for whom, as artists, my heart is warm, realizing, as I have already said, that there is usually some basic idea behind these statements. However, speaking collectively, I am forced to say that from the standpoint of the accepted use of scientific terms in science itself, all that can be said about many such statements is that they constitute a conglomeration of words which can be recognized as nouns, adjectives, verbs, etc., with the nouns in places where nouns should be, and verbs in the places where verbs should be. It has, however, been aptly said that it is not necessarily true that any conglomeration of words which is grammatically correct has necessarily any meaning.

The mechanism of the key action of a grand piano is a complicated affair,[31] but for the purposes of discussion it is necessary only to recognize one thing. If the key is depressed slowly, then for the whole of the time that it is being depressed it is in mechanical contact, through the mechanism, with the hammer. But even at the extent of its depression, the hammer has not yet struck the string. It results from this that the hammer is never in contact with the key at the instant when it does strike the string. Any departure from the condition associated with a slow depression of the key can only be one in such a direction as to cause the hammer to become free at an earlier period during the key depression, but it can never be in contact with the key when striking the string. The argument of the physicist is, therefore, to the effect that the condition of the hammer at the instant that it strikes the string is completely defined by its velocity, and that just as a baseball thrown at your head hurts to an extent depending on the speed with which it hits you and is otherwise independent of who threw the ball, or whether he gave it its speed in a long steady swing or a sharp rapid one, so the effect of the hammer on the string depends only upon its velocity at the instant of contact and is otherwise independent of who gave it that velocity or the manner in which he gave it. A velocity is a velocity and contains in its potentialities nothing about the history of how it was acquired.

Now, as a man of science who has some sympathy with artistic things, I do not receive this opinion of the physicist with delight, and indeed, I wish there could be some extenuating circumstances which would mean that it does not represent the whole truth. In meditating

[31] The key action of a Lester grand piano was on exhibit, from the Physics Section of the Franklin Institute Museum.

upon this matter, there is one possibility, one faint ray of hope, for those who claim that the player can control the tone quality independently of the intensity.

If we allow the hand to fall upon the key with a certain velocity, then the hammer starts to acquire its velocity at the instant that our finger touches the key and it continues to acquire velocity for a time thereafter, depending upon the mechanical properties of the various elements which enter into the construction of the key. Indeed, if everything were made of material which was absolutely rigid, and if the arm and hand were absolutely rigid, so that the motion of the hand was not impeded at all by its contact with the key, then everything would attain its final velocity at the very first instant of contact of the finger with the key. However, things are not made of material of infinite rigidity, so that there is a finite time during which the hammer is receiving its velocity.

Suppose now that I strike a key in the ordinary way and that, as a result, I give the hammer a certain velocity. Let me repeat this act, but before doing so, let me hold the key half way down. If, under this condition, my hand descends upon it, I can still arrange matters so that I give it the same velocity as before. I may have to allow my hand to fall from a greater height, but that does not matter, I can realize, if I wish, the same velocity. However, in this second case, I shall have given the hammer its velocity in a shorter time than that in which I gave it its velocity in the first case. In other words, the acceleration of the hammer head will be greater in the second case than in the first. Now the only reason why the hammer head moves at all is because it is connected to the shank, and the shank, in giving the head its velocity, has to bend. It is indeed by that bending that it succeeds in exerting a force upon the hammer. Now the force on a moving body is determined, not by its velocity, but by its acceleration. Thus since in the second instance which I cited, the acceleration was greater than in the first, it will be necessary for the shank to bend more in the second case than in the first. Now when the shank is bent in this manner, it possesses the power to bring about in itself vibratory motion. In other words, the shank does not rise up in a continuous uniform manner, but in a series of oscillatory jerks, and these oscillatory jerks will be greater in the second case which I cited above than they will in the first, even for the same average velocity at the end, because the shank was bent more in the second case than the first. Now it seems not outside of the realm of reason to expect that the type of motion set up in the string may depend not only upon the final average velocity of the hammer during these oscillations, but upon the extent of the oscillations themselves, and if such were the case there would be logical meaning to the possibilities of the player controlling the tone.

I have here, Fig. 12, a large scale model to illustrate this matter. It is purposely made rather flexible in certain of its parts so as to exaggerate the point I wish to illustrate. If I press A, which corresponds to the key, the hammer rises. At a certain point, the motion of the key is terminated by the stop S, but the hammer goes on. I will strike the key in such a manner that the hammer acquires its velocity during the whole range of descent of the key. You will observe that the hammer, after leaving the stop S, travels with a more or less uniform and steady motion.

I will now repeat this experiment, but under conditions in which the key is already depressed to a certain extent, and I will compensate

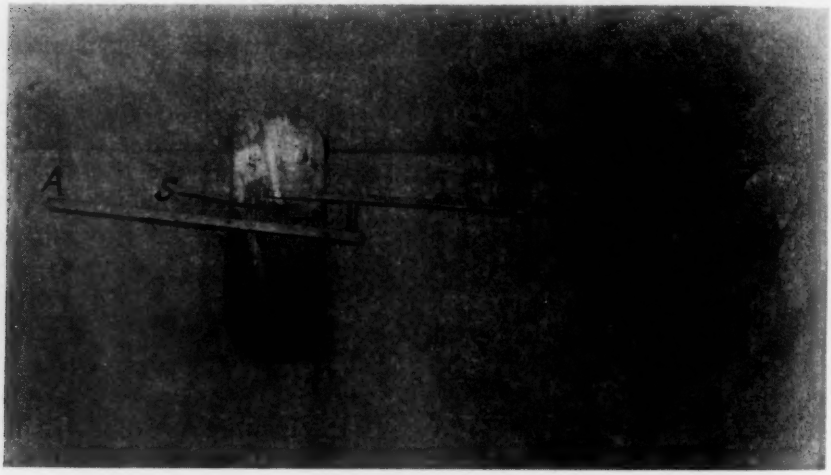

FIG. 12. In another form of this model the key was operated by the dropping of a weight. A spring could be incorporated, at will, in such fashion as to cause the acceleration to be reduced for the same final average hammer velocity. Thus two reproducible conditions were realized.

or this in such a manner as to create about the same velocity as before, but now in a shorter time. The shank associated with the hammer will be bent more and you will observe that it rises with an oscillatory motion superposed upon a steady motion.[32]

Now in the light of this experiment, the question which remains is whether such effects as these do actually play a part in practical performance upon the piano. We have with us this evening Professor Charles Weyl, who has initiated experimental tests on this matter at the University of Pennsylvania with the result shown in Fig. 13 (taken

[32] At this point the experiment was performed. The first striking of A resulted in a uniform rise of the hammer head, while the second striking resulted in a uniform motion with a number of oscillations superposed upon it. In the Appendix, Problem 3, I have given a calculation illustrating, in a general way, the degree of importance of the aforesaid oscillations.

from J. H. Jeans, "Science and Music," The Macmillan Co.). The figure shows examples of the comparison of the oscillograms obtained in two different tests. In each test, a note on the piano was played by a well-known piano virtuoso, and it was also played by letting a weight fall on the key. The upper curve of each test shows the result produced by the pianist, and the lower curve that produced by the weight. No difference is apparent.

The second example is different from the first in that the intensity of the note was different. Even with the simple apparatus which I have here this evening, we can see that the quality of the note depends

Hart, Fuller and Lusby

Sound-curves of a note on the pianoforte produced; (upper curve) by a well-known pianoforte virtuoso; (lower curve) by letting a weight fall on the key. The curves are exactly similar, showing that the virtuoso can produce no greater effect than can be produced by merely mechanical means.

Hart, Fuller and Lusby

A second pair of sound-curves produced respectively by virtuoso playing and dropping a weight on the pianoforte key. Again the two curves are seen to be exactly similar.

FIG. 13.

SOUND-CURVES PRODUCED BY HUMAN SKILL COMPARED WITH THOSE PRODUCED BY MECHANICAL MEANS.

upon the intensity.[33] Intimately related to this fact is the fact that if we play a note with the pedal down so that it continues to sound, the quality of the note itself changes as it dies down. However, the two notes played in different ways, but with the same intensity, show no difference in quality.

Figure 14 (taken from "Musical Acoustics" by Charles A. Culver, Blakiston Co., Philadelphia) shows what the physicist finds for two types of playing designated by the artist by the mysterious terms

[33] The apparatus referred to was the oscillograph and microphone. To illustrate the matter, a single note was struck with different intensities, and the wave form on the oscillogram was shown.

"cushion touch" and "rebound touch." Physics seems to know of no such touches, or at least of no difference between them.

Of course, an artist may claim that his ear is so sensitive that he can appreciate differences of quality which no scientific instrument can reveal. The answer of the physicist to this is that the physical apparatus is so sensitive that it can show very definite differences in quality in cases where the ear of the artist cannot appreciate any differences in quality at all.

I have here a mechanism which can be caused to play a note by allowing a weight to fall. The speed of the weight when it hits the key can be regulated by the height from which it falls.[34] Now by

FIG. 14. Recordings of piano tones. The upper trace was produced by a "cushion" touch; the lower by a "rebound" touch.

using this mechanism, I wish to strike a note several times with the key initially depressed to different extents, and with the height of fall of the weight correspondingly adjusted to give the same intensity in each case as judged by the ear. In this way we shall give the key, in different times, the velocity appropriate to the intensity realized; and if there is anything in the idea which I have mooted to the effect that the suddenness of impartation of velocity to the key affects the quality, we shall hear differences in the quality of the note when played under the various conditions cited. We will now make this test.[35]

[34] By a suitably chosen piece of rubber on the striking end of the weight, bouncing was avoided.

[35] At this point the test was made with no noticeable difference in the quality of the note as produced under the different conditions referred to.

Now I do not doubt, and physicists in general do not doubt, that the playing of some pianists can be more pleasing than that of others, but the general belief of the physicist is that in such cases, and apart from ordinary differences in musical interpretation, elements must become involved other than those concerned with mere tone quality. What elements suggest themselves in this connection?

The first thing which suggests itself is something having to do with the damper. After the note is played, and the finger is raised, the damper falls and terminates it. If another note follows after the damper has fallen, the two notes will be separated completely and such a situation, involving a succession of notes, constitutes staccato playing. On the other hand, if a certain note is played and the playing finger holds the key down until after the next note is played, both notes will sound together, and, in general, we shall obtain an undesirable effect unless for some reason or other this particular kind of effect is called for at the moment. Now between these two extremes, there is a borderline region of considerable subtlety, in which note No. 1 may sound for a very brief instant during the playing of note No. 2. Incidentally, I have already called your attention to the fact, and will again demonstrate it, that a note changes its quality as it dies down.[36] The superposition of what we may call the tail end of note No. 1 on note No. 2 may not produce the consciousness of the simultaneous playing of two notes, but just as a note and its overtones do not appear to the ear as individual entities, but weld into an effect characterized by quality, so the tail end of note No. 1, lasting for a brief instant during the playing of No. 2, may produce an effect which the ear would characterize in terms of the quality of note No. 2. If such an idea does represent the facts, we have a possibility wherein the skill of the artist has latitude for very refined and subtle development in producing a fairly wide gradation of effects which the listener might characterize in terms of tone quality. Again, if such idea were true, we should have another illustration of a case where the artist is ultimately right, but his reasons hopelessly wrong. You see how hard I am striving to find a place for genius in the technique of the artists. I hope they are duly grateful. I fear, however, they are only irritated.

I think another possibility exists when we have chords, in which different notes are sounded together, and the possibilities here exist even when the notes are separated, because an arpeggio is merely a spread-out chord, and the ear and brain possess the capability of remembering the various notes which have been played and contemplating them in a kind of instantaneous focus. However, I will confine what I have to say for the moment to chords. I had a rather interesting illustration of the matter concerned some time ago. I happen to con-

[36] Here the demonstration was given.

duct an orchestra and, being rather displeased with the first trombone on account of his failure to play with decision in a certain portion during a rehearsal, I evidently penetrated his conscience so much that he came up afterwards to apologize, and explained that he had been hesitant to play the particular passage concerned because, when he tried it, what he played sounded so terrible against the first horn. I was struck with an idea and asked him to come over with me to the piano. I played F and G together and asked him how it sounded. He said it sounded bad, and I agreed. I now played F, G, B, D, and we both agreed that it sounded well. (It was, of course, an inversion of the chord of the dominant seventh on G in the key of C major.) I next played the F and G again, without the other notes, and he confessed that it sounded better than it did before. "Yes," I said, "that is because you are remembering the other notes, which made the chord sound good."[37]

We cannot help observing the very strong bearing of this statement upon the ideas expressed in the above text. Incidentally, moreover, the second paragraph has a profound implication in relation to musical accents which will be well recognized by serious musicians. In a later portion of the letter in question, Franklin remarks:

"Farther, when we consider by whom these ancient tunes were composed, and how they were first performed, we shall see that such harmonical succession of sounds was natural and even necessary in their construction. They were composed by the minstrels of those days to be played on the harp accompanied by the voice. The harp was strung with wire (which gives a sound of long continuance) and had no contrivance, like that in the modern harpsi-

[37] Following the lecture, Dr. H. B. Allen called my attention to some remarks of Benjamin Franklin, whose versatility extended even to the expression of profound sentiments in relation to music. In a letter to Lord Kames ("The Life and Writings of Benjamin Franklin"—Smyth, Vol. 4, pp. 377–381), Franklin, after regretting what he regards as modern tendencies in music and the attempt to adorn natural beauty with what he considers inharmonious sounds, says:

"Give me leave on this occasion to extend a little the sense of your position, that "Melody and Harmony are separately agreeable, and in union delightful," and to give it as my opinion, that the reason why the Scotch tunes have lived so long, and will probably live for ever (if they escape being stifled in modern affected ornament), is merely this, that they are really compositions of melody and harmony united, or rather that their melody is harmony. I mean the simple tunes sung by a single voice. As this will appear paradoxical, I must explain my meaning. In common acceptation, indeed, only an agreeable succession of sounds is called Melody, and then only the co-existence of agreeing sounds, Harmony. But, since the memory is capable of retaining for some moments a perfect idea of the pitch of a past sound, so as to compare with the pitch of a succeeding sound, and judge truly of their agreement or disagreement, there may and does arise from thence a sense of harmony between the present and past sounds, equally pleasing with that between two present sounds.

"Now the construction of the old Scotch tunes is this, that almost every succeeding emphatical note is a third, a fifth, an octave, or in short some note that is in concord with the preceding note. Thirds are chiefly used, which are very pleasing concords. I use the word emphatical to distinguish those notes which have a stress laid on them in singing the tune, from the lighter connecting notes, that serve merely, like grammar articles, to tack the others together."

chord, by which the sound of the preceding could be stoppt, the moment a succeeding note began. To avoid actually discord, it was therefore necessary that the succeeding emphatic note should be a chord with the preceding, as their sounds must exist at the same time. Hence arose that beauty in those tunes that has so long pleased, and will please for ever, though men scarce know why."

Now, when you played this evening, the second trombone was absent, and he had a note which, if it had been played, would have made your note sound well with that of the horn." Having gotten this idea myself—for which, however, I naturally claim no originality, for doubtless all professional conductors know it better than I do—I started to experiment with it in other connections. It seemed to me that if, in the orchestra, there was a chord in which the flute had an important note and the horn had an important note, and the trumpet and trombone had unimportant notes, then if the flute and horn were weak while the trumpet and trombone were strong, the chord would sound wrong, even though all the notes were in tune. I found this to be the case, and frequently, when a chord in the orchestra would sound absolutely wrong as though wrong notes were being played, or as though notes were being played out of tune, it was possible to adjust it to sound beautiful simply by calling for an adjustment in the relative intensities of the instruments concerned in the chord. Moreover, there was a kind of bonus to these good intentions, for once the players had heard the chord as it should be, they did not have to remember about the intensity with which they should play, for their ears automatically sought, unconsciously, the right intensity by seeking more directly the attainment of the pleasing sound which, when once heard, represented the only goal which satisfied them.

And so I have wondered whether, in the case of chords played upon the piano, the pleasingness of the effect may not be determined in part by the relative intensities of the different notes played.[38] It is, of course, not to be expected that *equality* of intensity is the most desirable goal, but rather an adjustment of intensity between the different notes which is such as to have some relation to the importance of those notes in the chord. If the foregoing ideas are valid, it is probable that the unsophisticated listener would not attempt to analyze too closely in his mind the reasons why a chord played by one person sounded better than the same chord played with the same overall intensity by another. He would probably lump the whole impression together under the vague word tone, and say that one player had a better tone than the other.

Now in order to illustrate this matter, I have here a mechanism for playing four notes simultaneously with predetermined intensity. It is a horrible looking thing with steel fingers and I hesitate to let

[38] This matter has also been referred to by Charles A. Culver in "Musical Acoustics" (Blakiston Co., Philadelphia, page 109).

any artist see it. He would probably think it symbolic of my attitude towards his art. With this mechanism I will play a chord, and will repeat it several times with different relative intensities in the notes. I will leave to your judgment the question of how you would interpret the differences produced, whether in terms of tone quality or otherwise. However, it is to be remembered that you have been warned, and are

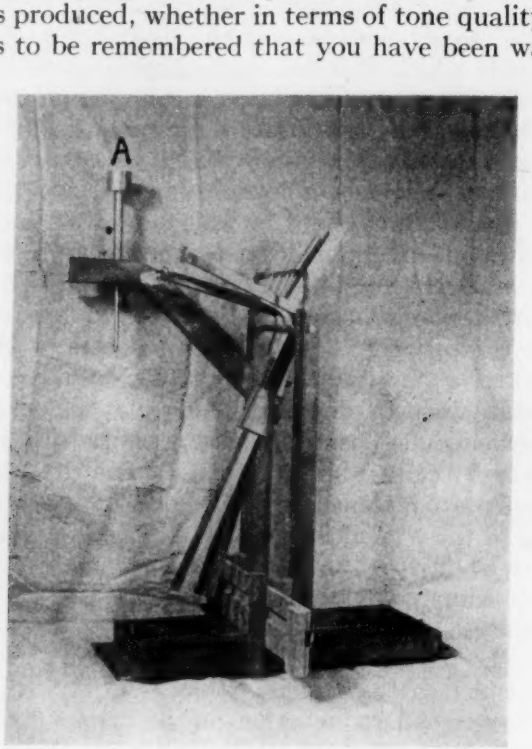

FIG. 15. The three "fingers" will be seen in the upper part of the figure. Their starting positions were determined by stops. Each finger was continued through a right angle bend into a rod which, with a weight at the end, constituted a pendulum. All three fingers could be released at once by burning a single thread. Bouncing from the keys was prevented by pins which were fastened to the ends of the pendulums and stuck into a board at the ends of the strikes. The part labelled *A* is the dropping weight mechanism referred to earlier, and incorporated for convenience as part of the present apparatus.

therefore less likely to accept a psychological counterpart of the physical fact as represented by mere changes in relative intensities of the notes than you would be if you were uninformed as to what is happening.[39]

[39] At this point the experiment was performed. Incidentally, the playing mechanism has some points of interest concerned with the problem of causing a single initiating act to result in a situation in which the different fingers strike their respective keys at the same time, but with different velocities. The mechanism is shown in Fig. 15 and its caption explains its operation. In the Appendix, Problem 4, the theory underlying the operation of the mechanism is given. In the case of this and other mechanisms used in this lecture, I am greatly indebted to the constructional skill of Mr. Frank Hamilton, the head of our machine shop.

I thought that it would be interesting for this audience itself to form some comparison between the sound produced by a falling weight and by an actual player. With this end in view, I will utilize the falling weight mechanism employed earlier. Four pianists have been kind enough to coöperate with me in this experiment. A screen will be placed around the piano, so that you cannot see what is happening. You will be handed cards. (The card is shown in Fig. 16.) On these cards are printed Case 1, Case 2, etc., up to Case 8, which correspond to the eight times which a note will be played. On the card you will find a place to record by a tick whether you think the note was played by a person or by the falling weight. After the experiment is completed, we will take a census of the results. I may say that since you will doubtless hear the falling weight mechanism, there is a danger that you will be biased. In order to avoid this, every time a person plays a note I shall let the weight fall in such a manner as not to play the note, and every time I allow the weight to fall so as to play the note, I shall see that a thump is made somewhere on the piano by a human hand, so that the mere noise of the mechanism will not give you any indication as to whether that mechanism is or is not responsible for the note.[40]

We will now put the results upon the blackboard, using the form of one of the cards, Fig. 16, to record them. The numbers represent the number of guesses corresponding to the places which they occupy

№ 129

	PLAYED BY	
	PERSON	WEIGHT
Case 1 . .		
Case 2 . .		
Case 3 . .		
Case 4 . .		
Case 5 . .		
Case 6 . .		
Case 7 . .		
Case 8 . .		

Name_____

FIG. 16.

№ 399

	PLAYED BY	
	PERSON	WEIGHT
Case 1 . .	* 110	14
Case 2 . .	35	93 *
Case 3 . .	79	75 *
Case 4 . .	* 108	50
Case 5 . .	91	74 *
Case 6 . .	* 59	102
Case 7 . .	* 77	80
Case 8 . .	54	96 *

Name_____

FIG. 17.

[40] At this point the test was made, and the numbers recorded in Fig. 17 represent the results.

on the card, and the star indicates in each test the guesses which were correct. Thus, in Case 1, the note was played by a person, and 110 persons guessed correctly.

It will be seen that in half of the cases (1, 2, 4, 8) the majority guessed correctly, and in the other half the majority guessed incorrectly. On the other hand, the total number of correct guesses is 692, as against 505 incorrect guesses. It can hardly be said that the experiment is very conclusive.

It is of interest to consider the data in another manner. If the guesses were determined simply by chance, as they would be if there were really no difference between the sounds produced by the weight and the persons, then a simple calculation shows that any one person performing the test of the eight cases, and then repeating the whole test a large number of times, would expect on the average to get all eight guesses correct once out of every 256 trials. Since there were about 200 people in the audience, there was reason to expect that perhaps one person would, on the basis of pure chance, have guessed all eight cases correctly. As a matter of fact, there were at least three people who had all eight cases correct. However, the numbers here involved are too small to provide any very reliable evidence one way or the other.

Perhaps the best way of making a test would be the following: We take an audience which meets weekly and comprises, say, 250 people. We ask the audience to devote five minutes to the test, and we pick out the people who have all eight guesses correct. Even if there be only one on the average in each meeting, we shall, at the end of a year, have a group of about 50 people who have made all guesses correctly.[41] Now if there is anything in the power of the ear to detect the difference between a weight and a person as regards the quality of tone produced, the aforesaid 50 people should form a very select group, and if we got them to meet as a special audience and repeated the experiment we have performed this evening, the results should be very conclusive in favor of the ability to discriminate between person and weight. On the other hand, if the selected audience is no better than this audience, one could hardly claim that the guesses were governed by anything other than the laws of chance.

APPENDIX.

(Problem 1.)

THE EFFECT ON INTONATION OF DEPRESSING THE STRING WITH THE FINGER.

Let the string of total length $l(=AB)$ be divided into two segments a and b by the point P, which is the point at which the finger presses the string down by an amount h to touch the finger board.

[41] Of course, if the audience is composed each time of the same people, and if one of these guesses correctly on several occasions, that will, in itself, be significant.

Consider a string of length l when stretched with the tension T, which is the tension applied before the finger is applied at P. We are interested in the note of frequency $(n + \delta n)$ given by the portion QB of the string as contrasted with the note of frequency n which would be given by the portion PB if the distance h had been zero. We have

$$n^2 = \frac{T}{4b^2m}, \tag{1A}$$

where m is the mass per unit length.

Differentiating logarithmically,

$$2\frac{\delta n}{n} = \frac{\delta T}{T} - 2\frac{\delta b}{b} - \frac{\delta m}{m}, \tag{2A}$$

where δT is the alteration in tension as a result of the depression h, δb is the alteration in length of PB, and δm is the alteration in mass

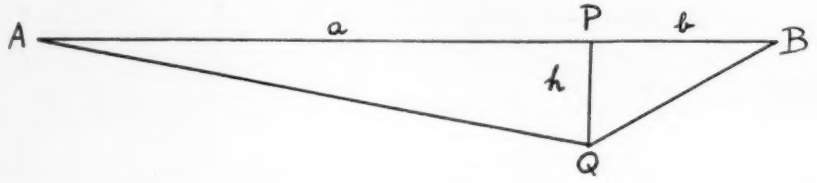

FIG. 1A.

per unit length. We have

$$\delta T = \left(\frac{\delta a + \delta b}{a + b}\right) Y\frac{m}{\rho}, \tag{3A}$$

where ρ is the density, so that m/ρ is the cross-sectional area. Y is Young's modulus. Also $T = 4b^2mn^2$.

Hence

$$\frac{\delta T}{T} = \frac{(\delta a + \delta b)Y}{4b^2(a + b)\rho n^2}. \tag{4A}$$

Also

$$\frac{\delta m}{m} = -\left(\frac{\delta a + \delta b}{a + b}\right). \tag{5A}$$

Hence

$$\frac{\delta n}{n} = \frac{1}{2}\left[\left(\frac{\delta a + \delta b}{a + b}\right)\left(\frac{Y}{4b^2\rho n^2} + 1\right) - 2\frac{\delta b}{b}\right]. \tag{6A}$$

Now

$$(a + \delta a)^2 = h^2 + a^2,$$

$$2\frac{\delta a}{a} = \frac{h^2}{a^2}.$$

Similarly

$$2\frac{\delta b}{b} = \frac{h^2}{b^2}.$$

Also

$$\frac{\delta a}{a+b} = \frac{1}{2}\left(\frac{a}{a+b}\right)\frac{h^2}{a^2},$$

$$\frac{\delta b}{a+b} = \frac{1}{2}\left(\frac{b}{a+b}\right)\frac{h^2}{b^2}.$$

Thus

$$\frac{\delta a + \delta b}{a+b} = \frac{h^2}{2ab} \tag{7A}$$

and

$$\frac{\delta n}{n} = \frac{1}{2}\left[\frac{h^2}{2ab}\left(\frac{Y}{4b^2\rho n^2}+1\right) - \frac{h^2}{b^2}\right],$$

$$\frac{\delta n}{n} = \frac{h^2}{4b^2}\left[\frac{b}{a}\left(\frac{Y}{4b^2\rho n^2}+1\right) - 2\right]. \tag{8A}$$

Suppose

$$\frac{b}{a} = \frac{1}{3}.$$

Suppose we are dealing with the 'cello A and the note being played is the second octave of this A, so that $n = 880$. Then since for steel $\rho = 7.5$ and $Y = 2 \times 10^{12}$ dynes/cm.^{2}, and also for the 'cello $l = 70$ cms., so that $b = 70/4$,

$$\frac{Y}{4b^2\rho n^2} = \frac{2 \times 10^{12} \times (4)^2}{4 \times (70)^2 \times 7.5 \times (880)^2} = 275.$$

Moreover, if subscripts s and g refer to steel and gut respectively,

$$\frac{Y_s}{\rho_s} = 10\,\frac{Y_g}{\rho_g}. \tag{9A}$$

Thus for gut

$$\frac{Y}{4b^2\rho n^2} = 27.5.$$

In both cases $Y/4b^2\rho n^2$ is much larger than unity, so that

$$\frac{\delta n}{n} = \frac{h^2}{4b^2}\left[\frac{b}{a}\left(\frac{Y}{4b^2\rho n^2}\right) - 2\right]. \tag{10A}$$

Moreover, if $b/a = 1/3$, then even for gut we may neglect the 2 in the preceding formula and write

$$\frac{\delta n}{n} = \frac{h^2}{16ab^3n^2}\left(\frac{Y}{\rho}\right). \tag{11A}$$

Incidentally, we notice from (4A) and (7A) that

$$\frac{\delta T}{T} = \frac{h^2}{8ab^3n^2}\left(\frac{Y}{\rho}\right). \qquad (12A)$$

(Problem 2.)

CONCERNING THE FORCES EXERTED BY THE FINGERS ON THE BOW.

In the problem which follows, the words "upwards" and "downwards," in relation to the forces, must be taken to apply to the components of the forces which are perpendicular to the string.

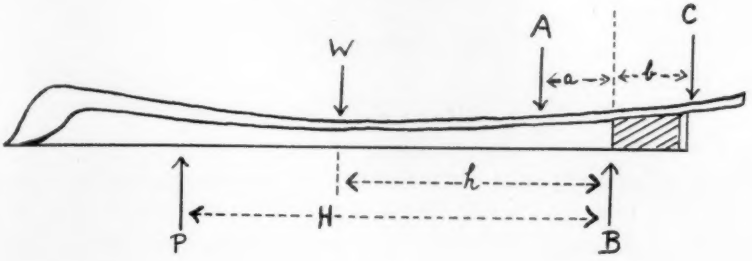

FIG. 2A.

Let P be the upward force on the bow due to the string.
Let A be the downward force on the bow due to the first finger.
Let B be the upward force on the bow due to the thumb.
Let C be the downward force on the bow due to the little finger.
Let W be the weight of the bow.
Let h be the distance from thumb to center of gravity.
Let H be the distance from thumb to point of contact with the string.
Let a be the distance from thumb to first finger.
Let b be the distance from thumb to little finger.
We desire to make P the same for all values of H.

We shall have two equations, one stating that the total force on the bow is zero and the other stating that the total couple about any point, say the thumb, is zero. With P and W given, these equations can determine two forces which will provide the desired conditions. Mathematically we can provide for the desired conditions with A and B alone, A and C alone, or B and C alone. We shall find, however, that if we try to get on with any pair, A and B for example, we shall have to have one of the forces, B, negative for certain points of application, and while we could secure this condition by cementing the bow to the thumb, so that it could pull downwards, the situation does not represent a practical one consistent with ordinary playing. In spite of this, we shall determine what A and B would have to be in order to secure the desired conditions even though, as will turn out, B has sometimes to be negative. Then we shall see how to make an adjustment, employing C.

Utilizing, therefore, only A and B, we have, since the total force on the bow must be zero,

$$P + B - W - A = 0, \tag{13A}$$

and since the total couple about the thumb must be zero

$$PH - B(O) - Wh - Aa = 0. \tag{14A}$$

Hence, from (14A)

$$A = \frac{PH - Wh}{a} \tag{15A}$$

and from (12A) and (15A)[1A]

$$B = W - P + \frac{PH - Wh}{a},$$

$$B = \frac{W(a - h) + P(H - a)}{a}. \tag{16A}$$

Thus A is positive if

$$H > Wh/P, \tag{17A}$$

A is negative if

$$H < Wh/P. \tag{18A}$$

Also

B is positive if

$$H > a + W(h - a)/P. \tag{19A}$$

Now in practice[2A] we always have $a + W(h - a)/P$ greater than Wh/P. Thus, from the point of the bow down to the value of H given by H_1, where

$$H_1 = a + \frac{W(h - a)}{P}, \tag{20A}$$

we have a positive A and a positive B, and this is all.

For values of $H < H_1$, we shall have a negative B and a positive A until we reach the value of H given by H_2 where

$$H_2 = \frac{Wh}{P}. \tag{21A}$$

However, the positive A and negative B will provide balance of couples. In order to bring B out of the negative range, we must now invoke C, but we must accompany it by a change of A from the value given by (15A) or otherwise we should destroy the balance of couples and the

[1A] We can add to these any C_0, δA and δB satisfying

$$C_0 + \delta A = \delta B,$$
$$a\delta A = bC_0,$$

but such addition is quite arbitrary.

[2A] $a + \frac{W(h - a)}{P} > \frac{Wh}{P}$ if $a > \frac{W}{P}a$, and this is always true if W is less than P, as it is.

balance of total forces. We can, in fact, have B zero in a limited region of $H < H_1$ provided that

$$P - C - W - A = 0, \tag{22A}$$

$$PH - Wh - Aa + Cb = 0. \tag{23A}$$

Hence, from (22A) and (23A)

$$A = \frac{P(H + b) - W(h + b)}{a + b}, \tag{24A}$$

$$C = \frac{W(h - a) - P(H - a)}{a + b}, \tag{25A}$$

and, of course,[3A]

$$B = 0.$$

The significance of the former limit H_2 has, of course, now evaporated on account of our alteration of A from the value given by (15A), so that we must seek new limits for the applicability of (24A)–(26A).

In the region concerned, where $H < H_1$, that is,

$$H < a + W(h - a)/P,$$

that is, where

$$H = a + \frac{W(h - a)}{P} - \epsilon,$$

where ϵ is a positive quantity, we have

$$C = \frac{W(h - a) - W(h - a) + P\epsilon}{b + a}. \tag{27A}$$

Thus C is always positive in the region concerned. However, A can become negative if $H < H_3$, where

$$H_3 = \frac{W}{P}(h + b) - b. \tag{26A}$$

There will be a positive value for H_3 if

$$\frac{W}{P} > \frac{b}{h + b},$$

and this will hold in general, since b is about 4 cms. and h is about 17 cms., and a value $P = 5W$ would be too high a value for practical usage.

[3A] Incidentally, we could make B positive if we wished by still increasing A by δA and C by δC, so that

$$\delta A + \delta C = B_0,$$
$$a\delta A = b\delta C,$$

where B_0 is the value of B desired. However, we do not need a finite value of B in this region.

For $H < H_3$ we must consequently increase B from zero, so that we shall not have to have a negative value for A. We can then set A zero and proceed only with B and C, according to the following equations:

$$P + B - C - W = 0, \tag{29A}$$

$$PH - B(O) - Wh + Cb = 0. \tag{30A}$$

Thus[4A]

$$C = \frac{Wh - PH}{b}$$

and[5A]

$$B = \frac{W(h + b) - P(H + b)}{b}.$$

Although A is zero in the above calculation, we can always add an additional A_0 provided that we add δB and δC, such that

$$A_0 = \delta B - \delta C,$$

$$A_0 a = b\delta C.$$

Thus, to sum up, we have as follows:
From the point to a value $H = H_1$ given by

$$H_1 = a + \frac{W(h - a)}{P}$$

we have

$$A = \frac{PH - Wh}{a},$$

$$B = \frac{W(a - h) + P(H - a)}{a},$$

$$C = 0.$$

[4A] C will be positive if $PH/Wh < 1$. Since we are in the region $H < H_3$, i.e., $PH < W(h + b) - bP$, i.e., in the region where $PH = W(h + b) - bP - \epsilon$, where ϵ is a positive quantity, we have

$$\frac{PH}{Wh} = \frac{Wh + b(W - P) - \epsilon}{WH}$$

$$= 1 - \frac{b(P - W) + \epsilon}{WH},$$

which is less than unity.

[5A] Since we are in the region (see footnote [4A])

$$PH = W(h + b) - bP - \epsilon,$$

where ϵ is positive, we have

$$bB = W(h + b) - W(h + b) + bP + \epsilon - bP,$$

so that B is positive.

From the value

$$H = H_1 = a + W(h - a)/P \quad \text{to} \quad H = H_3 = W(h + b)/P - b$$

we have

$$A = \frac{P(H + b) - W(h + b)}{a + b},$$

$$C = \frac{W(h - a) - P(H - a)}{a + b},$$

$$B = 0.$$

From the value $H = H_3 = W(h + b)/P - b$ to $H = 0$, we have

$$B = \frac{W(h + b) - P(H + b)}{b},$$

$$C = \frac{Wh - Ph}{b},$$

$$A = 0.$$

It is to be observed that the above represent the simplest forces necessary to secure the desired end. We can always add to them any forces A', B', C' which satisfy the two conditions

$$A' + C' - B' = 0,$$

$$A'a = C'b,$$

so that we can choose any one of the three arbitrarily and then calculate the other two.

(Problem 3.)

ENERGY IN THE OSCILLATIONS OF THE PIANO-HAMMER COMPARED WITH KINETIC ENERGY ASSOCIATED WITH THE AVERAGE VELOCITY OF THE HAMMER.

Let AB, Fig. 3A, represent the shank as bent during the stroke of the hammer, while the dotted line represents the undeflected line of the

FIG. 3A.

shank. Suppose, for convenience, that the shank is of square cross-section and let: M = mass of hammer; n = length of shank; d = depth

of shank; b = breadth of shank; Y = Young's modulus of elasticity of the material of the shank; y = maximum displacement of hammer head in relation to its position with unbent shank, as represented by the dotted line; α = statical force per unit displacement of head when end B of shank is held fixed.

If T is the time to impart a velocity v to the hammer head, the average force F during the impartation of this velocity is given by

$$F = Mv/T.$$

In order to supply this force, the hammer must be bent to an extent y given by

$$y = F/\alpha = Mv/\alpha T.$$

The statical energy in the shank due to bending is $\alpha y^2/2 = M^2v^2/2T^2\alpha$ for the displacement y, and this quantity is of the same order of magnitude as the maximum kinetic energy k in the *vibrations* of the hammer. Thus, as regards order of magnitude,

$$k = M^2v^2/2T^2\alpha.$$

The kinetic energy associated with the motion of the hand with velocity v is K, where

$$K = Mv^2/2.$$

Hence

$$\frac{k}{K} = M/T^2\alpha.$$

Now the theory of elastic bending of beams gives

$$\alpha = Ebd^3/4n^3.$$

Hence

$$\frac{k}{K} = 4Mn^3/T^2Ebd^3.$$

To take a numerical example, if M = 10 grams; n = 15 cms.; $T = 10^{-2}$ sec.; $E = 1.5 \times 10^{11}$ dynes/cm.^{2}; $b = d = 0.5$ cm., we have

$$\frac{k}{K} = 2.7 \times 10^{-3}.$$

Thus, in the case cited, the kinetic energy in the oscillations would be comparable with 0.3 per cent. of the total kinetic energy of the hammer head, and hence is quite small.

(Problem 4.)

THEORY OF THE PIANO KEY STRIKER.

The purpose of the device is to secure a condition in which a single release of the mechanism causes all the keys to be struck at the same

instant, and with a velocity ratio which is assigned. Each key striker amounted in principle to a pendulum with a bob B whose position could be adjusted, and with an upper part perpendicular to the length of the pendulum and terminating in "fingers" F and G, only one (F) of which was employed for striking.

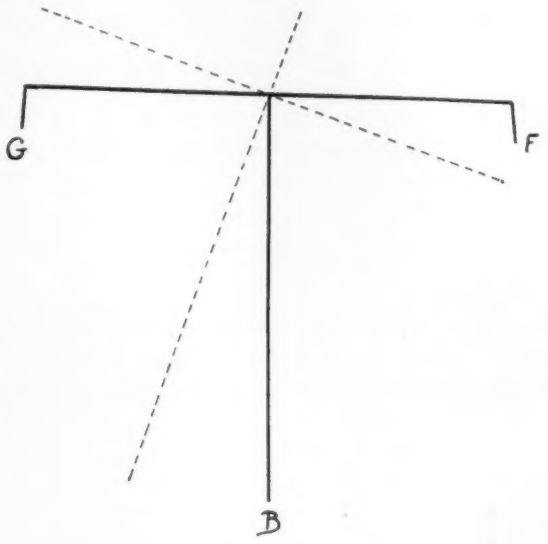

FIG. 4A.

Now considering two of such devices as the above, it is our problem to arrange that they shall hit their respective keys

(1) at velocities in a preassigned ratio η,

(2) at the same time t after release from their initial angular displacements θ_{01} and θ_{02} measured from their positions of equilibrium under gravity alone.

Denoting the two pendulums by subscript 1 and subscript 2, let us write

$$\theta_1 = \theta_{01} \cos \omega_1 t, \tag{31A}$$

$$\theta_2 = \theta_{02} \cos \omega_2 t, \tag{32A}$$

where θ_{01} and θ_{02}, as above stated, are the angular displacements initially and t is the time of contact of the "finger" with the key, this time being the same in both cases.

The angular velocities at key contact are $v_{\theta 1}$ and $v_{\theta 2}$, given by

$$v_{\theta 1} = -\omega_1 \theta_{01} \sin \omega_1 t, \tag{33A}$$

$$v_{\theta 2} = -\omega_2 \theta_{02} \sin \omega_2 t. \tag{34A}$$

The ratio $\eta = v_{\theta 1}/v_{\theta 2}$ is the quantity assigned, and is

$$\eta = \frac{v_{\theta 1}}{v_{\theta 2}} = \frac{\omega_1}{\omega_2}\frac{\theta_{01}}{\theta_{02}}\frac{\sin \omega_1 t}{\sin \omega_2 t}. \tag{35A}$$

Now, the contact of finger and key determines in each case the same value[6A] of θ, so that $\theta_1 = \theta_2$. Hence, from (31A) and (32A),

$$\frac{\theta_{01}}{\theta_{02}} = \frac{\cos \omega_2 t}{\cos \omega_1 t}.$$

Thus, from (35A)

$$\eta = \frac{\omega_1}{\omega_2}\frac{\cos \omega_2 t}{\cos \omega_1 t}\frac{\sin \omega_1 t}{\sin \omega_2 t} = \frac{\omega_1}{\omega_2}\frac{\tan \omega_1 t}{\tan \omega_2 t}.$$

Hence,

$$\eta = \frac{\omega_1 t}{\omega_2 t}\frac{\tan \omega_1 t}{\tan \omega_2 t}. \tag{36A}$$

Suppose we now arbitrarily assign ω_1 and θ_{01} to give a certain desired intensity. Since $\theta_1 (= \theta_2)$ is assigned by the geometry of the mechanism in relation to the keyboard we can determine $\omega_1 t$ and so t from (31A). We now assign η the desired value, and determine $\omega_2 t$ from (36A). We now determine θ_{02} from (32A), observing that $\theta_2 = \theta_1$.

In practice we need not proceed by measurement, as implied above, but by trial. The main purpose of the discussion is to show that it is *possible* to realize the conditions desired, viz., simultaneous time of striking, and assigned velocity ratio.

[6A] It is not *necessary* to provide for this condition. We can allow θ_1 and θ_2 to have different values, and incorporate the ratio θ_1/θ_2 in our calculations.

"BENJAMIN FRANKLIN, INTERNATIONAL PATRIOT" *

BY

SIR LOUIS BEALE, K.C.M.G.,

Formerly Co-ordinator of Empire and Allied Requirements, British Supply Council in North America.

LADIES AND GENTLEMEN:

Honored as I am by the invitation to address this august assembly, I have an uneasy feeling that your choice was, to say the very least, hazardous. To quote Franklin, "The worst wheel in the cart makes the most noise." I am not a historian, as I fear will be very apparent in the next few minutes. I am not a scientist, as you will soon discover. I am not even a politician, as I hope will be obvious to all of you. But I am a humble student of human nature, almost by profession. It has long been my business to watch the reactions of men and women to new ideas. I am thinking of my twenty years in the Far East, when "East met West" and often failed to appreciate each other, and I amuse myself sometimes by wondering how men of other days would have reacted to the stresses of modern life. With your permission, I propose this evening to lead your thoughts along that same path, to try to imagine how Benjamin Franklin would have thought and acted if he had been alive today, if he had been studying human nature today, if he had been confronted with the problems which today harass every one of us.

To understand Benjamin Franklin I do not go to the historians so much as to the autobiography. In these days of debunking I hardly dare quote a biography. The heroes of our generation seem to have become the targets of the new. But Franklin's autobiography shows the man as he really is—or as he would like to appear to posterity. As a matter of fact, Franklin impresses me with his candour. He sets down his *errata*, his mistakes, with an engaging lack of pride. Besides, he was a craftsman—always a wholesome check on self-deception. And he learned about life the hard way, and so achieved a certain directness of thought. Altogether I am inclined to believe that the autobiography gives a fair picture of the man's mind, even if his judgment is not always justified in the light of the history of the last hundred years. He thinks, for example, that if the Colonies had raised their own forces to fight the Indians, and had paid for them, the revolt against English rule might have been averted. Personally, I believe that the revolt

* Presented at the Annual Meeting of The Franklin Institute, held in the Lecture Hall, Wednesday, January 17, 1945.

was inevitable, largely owing to economic causes, and that it has resulted in benefit to England, to the American people, and indeed to the whole world. But those deeper economic causes were hardly visible to the men of the period. It is not that his judgment was not sound, but rather that he was reasoning from insufficient premises.

It has often been said of Franklin that he was the typical American. A charming compliment to the Nation! But, one moment! Please remember that he was an Englishman for many years of his life. I rather feel inclined to say that he was a typical Englishman. You will remember that the American Colonists always claimed their rights under Magna Carta (which, like another famous and more recent document, was never signed) when questions of taxation arose. "No taxation without representation" is an English phrase, as is "What touches all must be approved by all." These sayings were at the root of all Franklin's sturdy political opinions—typically English.

But let us return to the man and his qualities, regardless of nationality. I should put first his pluck. Fancy leaving Boston at the age of 17 to look for a job as a printer, in Philadelphia. I make no rash comparisons between Boston and Philadelphia in 1722! There may be Bostonians present! But consider the journey and the difference in religion and background between the people of the two cities! And think also of his toleration! He goes also to New York and London when still a youth, and makes no unfair comments on any of these places. Even then his forbearance and understanding were noteworthy. To me that is the beginning of the international spirit, a spirit of tolerant comprehension, based on a profound knowledge of human nature.

Hot Springs, Bretton Woods, Dumbarton Oaks, Conferences in London, Conferences in Washington, missions to China, what use are these unless the peoples of the world are animated by the spirit of mutual trust? of mutual forbearance? In very truth we nations are "islands shouting to each other over seas of misunderstanding." Even your country and mine, united though we now are, in our struggle against common enemies, must guard against any lack of that mutual trust and forbearance which will be so vital if we are to win the peace as we are going to win the war. May we pause for a moment to consider this question a little more closely?

It has often been said that we British and Americans have additional trouble in understanding each other because we only *seem* to speak the same language. I know it is said half in joke, but the matter is so vital to the future of the whole world that we might analyze the point a little further. I do not worry so much about words with directly opposite meanings. For example, we British "table" a document for further discussion. You "table" it when you put it away. But take the question of speech and rhythm. An Englishman may sound lazy,

offhand, or even arrogant, to an American. An American may sound loud and dictatorial to an Englishman. Of course, we should both deny the accusation that we individually either give or receive such impressions. But I am quoting the results of research by Professor Lloyd James, a phonetician who knows both countries intimately. If his ideas are correct, we have an opening for misunderstanding here.

I said just now that I was not concerned with words with contradictory meanings. Something far more serious are the words with slightly different connotations. What is a Liberal in this country, and in Britain? What is a Socialist, in this country, in Britain, in France, in Italy, in Greece? I do not ask the question about words like Communist or Fascist, which have become merely terms of abuse.

Every time I talk about the British Cabinet in this country, I feel that I may be misunderstood. Every time anybody mentions President Roosevelt's Cabinet, I have to make a conscious effort to realize what the term connotes. To a Britisher, a Cabinet is a body of men and women, most of whom have been elected by their constituencies, each of whom is responsible for the affairs of his department and must answer in parliament for the administration of that department. Collectively they are responsible to parliament for the policy of the Government and a majority in Parliament can at any moment insist on a new election, which may involve a complete change of ministers. As far as I know, the word Cabinet brings up to you ladies and gentlemen a very different picture. For you differentiate between the legislative and executive functions of the Central Government to an extent which is foreign to British practice.

Now let us come to a far more serious question. What do we mean by the word "democracy"? There is a tendency to apply it to any of the United Nations, regardless of their form of Government. I name no names. There is again even a tendency to suggest that the return of kings to certain small countries in Europe is "undemocratic," regardless of the desires of certain peoples, such as the Norwegians, who wish to have their king back on his throne.

Is it any use under modern conditions to be satisfied with that great historic phrase, "Government by the people for the people"? I fear not.

In the Greek City State when all the free citizens could be summoned by the Herald's trumpet to the Assembly in the Market Place, it was possible to talk of government by the people. But today everything is on a different scale. In a modern democracy all we can expect is that the will of the people shall in the end prevail, that they should as far as possible choose the sort of people that they prefer to govern them, that they should possess some rapid means of controlling the officials who carry out the orders of the elected representatives, and that they should respect the rights of minorities.

Do you feel that this is a weak alternative to the totalitarian or the

communist State? That I am whittling the sacred term "democracy" down to a shred of its former proud grandeur? My answer is one that I believe Franklin would have given if he were alive today.

You notice that I used the phrase, "the will of the people shall in the end prevail." I believe that Franklin would have agreed. We all know the sneer, "The people are not educated enough to know what they want." Had all the Colonists of 1775 enjoyed a High School education? How many printers were there then in New York? And yet those people knew what they wanted—freedom from oppression and injustice, freedom to manage their own affairs. They chose the sort of men to rule them that they wanted, the sort of men who drafted the Declaration of Independence.

I venture to believe that the same is true today. England, the U.S.A.—and Germany—have to rule them the sort of men they want. We want freedom, the Germans wanted a master. I admit that Hitler gained power by force and fraud, but the people acquiesced. And they shall have their reward. And we shall have ours.

Yes, gentlemen, I really trust the people as Franklin did. But you remember his leather-apron club, the Junto, from which developed the circulating library. I am prepared to allow the people to choose their leaders as they have done in the past, and to allow the leaders to frame the detailed and complicated schemes that modern conditions demand—with Press and Radio as watch dogs. But I hope the leaders will be educated and will come from all classes of society.

By leaders I do not mean the rich and the privileged or the sons and daughters of the rich and privileged. Nor did Franklin. Actually it was in 1745 that Franklin put forward his "Proposals for establishing an Academy" which, as you well know, was constituted in 1749. It was not until about 1870 that an English Minister, after a further extension of the suffrage, uttered his celebrated jibe, "Now educate your masters," a jibe that contained a great truth. Franklin's Academy as originally planned was necessary for a true democracy in the makign. Benjamin was preparing to train the leaders, not only the children of the privileged classes, for whom Latin was the staple of the curriculum. Franklin advocated straightforward English and the like as the foundation of the education at the Academy. Clearly he was prepared to see the leaders come from the people.

Can we claim that Franklin foresaw another of the great problems that confront the democracies of today,—the need for the control of the officials who constitute what is termed today the bureaucracy?

Franklin fought officialdom with publicity from his boyhood. But he was not faced with the entrenched armies of officials we find today. It may be that he was too honest and efficient an official himself to forsee the danger. I cannot imagine him increasing his staff so as to increase his own importance. As Deputy Postmaster General he would

never have converted a deficit into a credit if he had employed such methods. Of course in those days there were no typewriters for writing letters in triplicate and so increasing correspondence and staff to deal with it. There were then no telephones to interrupt logical thought and encourage pointless gossiping. If Franklin had been a true bureaucrat he would never have published in 1787 "Observations on the causes and cure of smoky chimneys." Had he been a bureaucrat, he would have agitated instead for a new agency under his control, the "O.C.C.S.C."!

I wish Franklin had continued his autobiography to cover his life in France, where he must have seen officialism and favouritism gone, not just wild, but mad, stark mad. He would have probably made the necessary pungent criticisms and foreseen that his own country, whose colossal expansion he did forsee, would encounter the same difficulty though in a different form.

As an ex-official I know of no problem more urgent than that of teaching officials that they are the servants, not the masters of the people. The larger the community, the greater the difficulty. I almost said "the younger the official the greater the difficulty." I am not prepared to say that all the newly appointed should be stigmatised as

Sons of Belial
High flown with insolence and wine.

They are often earnest young men, slightly overcome with a sense of their own knowledge, and intoxicated by the prospect of power. But none of us are prepared to accept a dictatorship of the bureaucracy and I am convinced that we must face that danger. I see no chance that, for years at any rate, there will be any diminution in the importance or the power of the Central Government of any country. In the present state of the world I should indeed view it as a calamity. The really democratic countries, in the true sense of the word, are the small nations, Norway with its king, Switzerland with its republican President. I would even be prepared to argue that Britain is more democratic in its method of government, notably in the control of her officials, than the United States. But, to avoid fruitless discussion, I will pass over this point. What I do maintain is that we, the greater democracies, must maintain a constant watch against this danger. We must be on our guard against slipping backwards towards a new form of tyranny. Life, the life of a nation, is a "becoming," not a "being." As the Greek Heraclitus said two thousand years ago "Everything is in a state of flux." Heraclitus knew it, Franklin knew it and lived it. We, even more than our fathers and our grandfathers, must recognize it as a profound truth. Let us boldly admit that officialdom is a necessary concomitant of the modern state and organize accordingly—organized I trust in terms of an irreducible minimum, constantly reduced still further!

"They thought they were making a paradise and found it was only a parade ground" may be the epitaph of some of our young bureaucrats. It is for us to maintain constant vigilance—the eternal price of freedom.

One modern problem Franklin did have to face, almost in its modern form,—the treatment of a minority, the Empire Loyalists, those colonists who retained their allegiance to England. I need not tell you his attitude towards what was a truly thorny question.

We have the same problem today, both in our own countries and in those which are being liberated, thanks to their own and our united efforts. How far should a democracy give to minorities complete freedom of speech, freedom of assembly, freedom to carry arms? How far can a democracy be trusted even in its legislation to pay due respect to the claim of a minority that it shall not be unduly oppressed by the wishes of a majority?

Our problem is complicated however by modern conditions. Democracy has enemies to face who possess new weapons. War does not begin nowadays with a declaration of war, or even by an overt act of violence. It begins by an infiltration of ideas, more poisonous and effective than poison gas, into the enemy's country. In the old days I would have said cheerfully "*Magna est veritas et praevalebit.*" "Great is truth and it will prevail." What would Franklin have said in these days of Radio and multiple printing presses, of bombs and tommy guns, of professional revolutionaries, of young men and women who have spent years of their lives in underground warfare and have learnt to consider human life of scant importance?

For our democracies I would remain firm in my old belief in the liberty of the press and of the spoken word. For Europe immediately I am not so sure. Think of all those tortured countries, bereft of their natural leaders, torn by dissensions, uncertain of the path they would wish to tread, their minds racked by doubts, their bodies and minds weakened by hunger and cold. We cannot even imagine what their scale of values is today, what they are prepared to barter for the satisfaction of their immediate needs of food and clothing. The British may have some idea, for they too have suffered physically. But in this country we can hardly hope to understand their mental attitudes, towards collaborationists, towards their old rulers, towards escapees, towards the allies who are bringing them freedom from their oppressors. Can we here understand them? Perhaps, but probably not. Pity them? Yes. Condemn them for folly? Never. But should we give them freedom immediately? Freedom to carry arms? Freedom of Speech? Freedom of the Press? Europe may not be ready for immediate and absolute Freedom which might degenerate easily into anarchy.

Only the bond are free.
Only the free are bond.

There is no real freedom unless men are ready to submit to certain restrictions which ensure freedom for the many. Anarchy is not freedom. It is bondage. It is only the free that can enter into a bond. The unfree are those who are coerced into obedience by the stronger, not necessarily the majority. Perhaps the hardest lesson for a majority to learn is to respect the rights and opinions of a minority. When that lesson has been learnt, true democracy begins. I am not prepared to admit blindly that every one who has taken up arms for his country is a true democrat or a true patriot.

You see, ladies and gentlemen, whither the argument leads us—to the definition of a patriot. I do not hold with Dr. Johnson and his "Patriotism is the last refuge of a scoundrel." I even prefer, though I dislike, the slogan "My country, right or wrong" at all events in time of war. What I do believe in is the patriot who supports his country even when changes occur which affect him to his own disadvantage. To take one single example, a member of the House of Lords in England, who lost so much of his power in 1911 and loses none of his patriotism, is a patriot.

The patriot today may have to face changes in the political and economic life of his country—as Franklin did. He may put forward alternative policies—as Franklin did. But when he sees the changes are inevitable, he will accept them fully—as Franklin did. When war burst forth between England and her rebellious colonies, Franklin offered his property to the revolutionary Government. "Their lives, their fortunes and their sacred honour." What a magnificent phrase!

Are we to expect such a cataclysmic experience as that which overtook Franklin? Have we to face an engulfment of old traditions, a reversal of values? Is our patriotism to be put to such a test?

Some times I wonder. I think of Britain, her foreign investments liquidated, her foreign trade almost destroyed, her towns and factories decimated. Will she recover? Undoubtedly, given time and hard work. But what a test of patriotism! I trust her people, their patriotism. I trust them all, the poor and what remains of the rich. But what if her leaders in commerce and industry were to say "The more we work, the more we shall be taxed. Why work?" And taxed they will be for years and years. There is foreign trade to be recovered, there is the carrying out of the Beveridge Report, or its equivalent, to be paid for, the towns to be rebuilt. No, they will work. I trust them. The privileges of rank have faded. The privileges of wealth are passing. It will be a new Britain to which Britons will owe and give their loyalty.

Will you, ladies and gentlemen, have the same problem to face? Loyalty to a country whose social and economic structure will have been transformed, not always to your personal advantage? Will the rich here have to face the problem, that the rich are facing in Britain?

Will your patriotism which has shone so brightly in war be as highly tested in peace? It is not for me to answer that question in respect to your country. But I know that Franklin would have faced it and I know the answer he would have given. In times such as these, his common sense would have shone forth as glorified common sense. He was the true patriot.

You may remember that the title of these remarks of mine was "Franklin, the international patriot." We have considered the test of patriotism, an active love of country that will survive revolutionary change. May I carry you further to the word "international"?

I don't know whether the word "international" has to you a pleasant flavour. To those of you who are historians or scientists, it will not be disagreeable—except perhaps for a slight prejudice against translating from a foreign tongue the work of some of your foreign colleagues. Jealousy? No. Just honest friendly competition.

What did Franklin do with his remarkable discoveries in Physics? He sent them direct to the Royal Society in London for discussion, criticism and possible eventual publication. Could anything be more internationally minded? It is true that scientific honours poured upon him later, from England, France and elsewhere, but he took the initiative.

After this war what would Franklin have done? Surely he would have done all in his power to restore international scientific cooperation.

I do not propose to outline the methods by which this can be achieved, except to say that first and foremost the old lines of international scientific communication must be cleared of obstructions, the old Universities in liberated countries restored to their former efficiency with the necessary equipment and libraries.

There is no need for me to stress in this assembly the need for pure as well as applied science, for the search for truth for its own sake. If it were I should merely recall the importance of Franklin's work in the development of modern electrical practice. If I wished to prove the international character of such work I should enumerate the names and nationalities of say six leaders in physical research, American, English, French, German and Italian.

Mark you, I don't expect any higher standard of a scientist than of anybody else and I don't believe that it will ever be possible to expect them to refuse to fashion new weapons. (I am thinking of robot bombs that will cross the oceans) if they are asked to do so by their governments. Why should the scientists be expected to pull themselves up morally by their own boot straps when the rest of us cannot? For, obviously, what the world needs is better men and women, more love, more pity, more loving kindness. Why expect scientists to be any better than the rest of us,—any less foolish, less greedy or short-sighted? When the world renounces war, the scientists will cease devising

instruments of war. Until then, we must judge them as ordinary human beings.

Franklin did not have to face all the problems that confront us today. Science had not then been harnessed to totalitarian warfare. To him science was rather a bond that brought together men interested in the advance of pure knowledge, men of all nations. What would he have thought of international cooperation in the political field as we are envisaging it today? Cooperation with the definite object of preventing war in the world? I would suggest that his conception of a future British Commonwealth of Nations indicates his probable attitude towards such an organization devoted to world peace.

Let me quote his "Observations" written in 1751:

"There are now supposed to be upwards of one million English souls in North America (though 'tis thought that scarce 80,000 have been brought oversea). This million, doubling suppose but once in 25 years, will in another century, be more than the people of England, and the greatest number of Englishmen will be on this side of the water. What an accession to the British Empire by sea and by land. What increase of trade and navigation. What numbers of ships and seamen."

And again:

"I have long been of opinion that the future grandeur and stability of the British Empire lie in America."

But don't imagine that he ever failed to attack those who would keep the "Colonies" in subjection.

Said he "These men make a mighty noise about the importance of keeping up our authority over the Colonies. They govern and regulate too much. . . . The true art of governing the Colonies lies in a nutshell. It is only letting them alone."

England has learnt the lesson. It is for that reason that I talk with such pride of the British Commonwealth of Nations—Canada, Australia, New Zealand, South Africa.

It is true that Franklin foresaw the growth of the United States to its present position as the strongest power in the world and hence the natural center of the Commonwealth. If his ideas had been accepted —but no, it is not worth while to pursue that line of thought. They were not accepted and progress has been in a different direction. I say progress, because we have no proof that this country would have achieved what it has achieved if it had remained within the British Empire. We cannot even be certain that the British Commonwealth of Nations would have reached its present development.

I make no apology for citing the free British Commonwealth of Nations as the greatest example of practical internationalism that the world has seen, simply because the idea of war between them is impossibly unreal. It just couldn't happen. You may challenge me with Eire. I admit that that island might conceivably ally itself with an

enemy of Britain, even though the number of Irishmen today fighting in the British armed forces is fantastically high. The reason of course is that the English have treated the Irish badly in the past, that a tradition of friendship and of common interest has not been built up during the last few hundred years. We have treated her, I feel, with extraordinary generosity during the present war and doubtless we shall get our reward in the future. I admit that at the moment I should have preferred something a little more tangible.

My point, of course, is that the international spirit can only be built on a basis of friendship and common interests, mutual understanding and mutual forbearance. Our problem today is to build up that friendship based on common interest, mutual understanding and mutual forbearance.

It is once more a question of enlightened common sense of which Franklin was so wonderful a master.

How do you think that it was that Franklin achieved such a triumph in Paris, both diplomatic and social? When I think of the loans he squeezed out of the French Government, I am lost in wonder. My feeling is that he remained himself—a true American. I don't suggest that he didn't nurse his popularity with some care—always wore a fur cap so as to be distinctive. You may say that that is typically American. My point is that a man need not be cosmopolitan to be international. Franklin understood the French, he bore with their failings, he admired their virtues,—but he remained American.

We are all agreed that we don't want another war and are making plans accordingly. But we know the plan will not fulfil its purpose unless there is mutual understanding and forbearance behind it. We know too that disagreements between nations often arise from quite small causes, often economic, and are fomented by further minor differences. Clearly the overall peace plan is not enough. We must endeavour to solve difficulties, notably economic difficulties, as they arise. Otherwise we are not smoothing the path of Peace. Again a matter of common sense coupled with some knowledge of human nature. Franklinesque, to coin the necessary word.

Among the most successful of the Committees convened at Geneva by the League of Nations was, I believe, that on Distemper among dogs. All the members cared about dogs and their ailments. There were no international complications, no national jealousies. The results were good for the dogs and good for international understanding. I am suggesting by this tiny example that international committees which deal with subjects of common interests do much, in a democratic world, to help the nations to understand each other. Frankly, I don't believe much in "Getting together" just for the sake of getting together. I am not even enamoured of Olympic Games from that point of view. The

common interest there is beating one another, and the methods adopted have not always been very clean or conducive to real friendship.

What seems to me ideal is the gathering of scientists from every nation to plan for the common good. Take for instance Agriculture. Franklin was always trying to do something for what he felt was the basic concern of mankind and the most honest way of making a livelihood. He would go to endless trouble in sending friends seeds and cuttings from abroad and suggesting new methods of cultivation. He was, I believe, the first to suggest crop insurance. How he would have delighted in an international body devoted to research in agriculture, to exchanging knowledge, to helping other nations to increase their output. His mind would rise far above the folly of imagining that one nation could be prosperous in the midst of an impoverished world, a fallacy that I fear is not entirely dead today.

Let us quote Franklin's own words and actions. "There seems to be but three ways for a nation to acquire wealth. The first is by War, as the Romans did by plundering their conquered neighbours. This is Robbery. The second by commerce, which is generally Cheating. The third by agriculture, the only honest way, wherein man receives a real increase of the seed thrown into the ground, in a kind of continual miracle, wrought by the hand of God in his favour, as a reward for his innocent life and his virtuous industry."

While Franklin was in France, he procured American seeds for the *Jardin des Plantes*. The royal physician from Vienna wrote to him about the diseases of wheat. He gave information about the construction of a drill plough. He was asked how to protect fruit trees from frost. He discoursed on the value of controlling insects, befriending birds as the destroyers of pests. He was interested in the culture of hemp, flax and silk, believing that these crops had great possibilities in America. In England he gathered information on silk culture. He wrote from that country concerning alfalfa, then called lucerne, and introduced rhubarb and scotch kale to America. He sent also seeds of turnips, cabbages and peas. He procured for friends abroad scions of the Newtown pippin apple and hickory nuts, walnuts and chestnuts from this country. Why, the man was an international Institute of Agriculture in himself!

Do you wonder why I harp so on agriculture? I will tell you. Because of the two billion people in the world today, at least a billion and a half are hungry. A billion and a half don't get enough to eat. I don't mean that they are all starving to death. I mean that they are hungry, they don't get enough to satisfy their natural desire for enough health-giving food. I should be sorry if you were not to believe me. It would only show that you don't realize the facts. Remember also, I am not talking about war conditions. I am talking about times of peace.

I don't like the thought of a billion and a half people going hungry. None of us do. Probably that is why we don't think about it and forget about it so quickly. But now that we are thinking about building a brave new world, we should do well to face it.

There was a time not so long ago when food was permanently short, even in this country and England. We used to talk about the "hungry forties." Starvation was very close, even in countries that imagined themselves civilized and advanced. You remember the gloomy predictions of Malthus early in the nineteenth century, that with the growth of population there would not be enough food in the world for all of us. I admit that his followers made some absurd deductions, but Malthus was right in the light of the knowledge then available. What happened? Today we here have enough to eat and this continent can export enormous supplies to other countries. Most of your population are, indeed, engaged in manufacture and commerce. Only 23 per cent. remain in agriculture, a fact that would have astonished Franklin profoundly. It might have even caused him to be slightly less impolite to commerce as a means of earning a livelihood. For Franklin was essentially a pragmatist. If he saw anything working successfully, he would have seen the good in it.

The answer to our question is of course "Science." Not just the inventions of practical men, such as the wheel plough or the seed drill but the application of the results of pure scientific research to agriculture. Let us take 1840 as our starting date.

Since that time agriculture in certain countries has been transformed, to an extent unnoticed by town dwellers. New seeds, less animal diseases, better fertilizers, more farm machinery, more irrigation, better marketing, better transport, more bread, more meat, more fruit, more milk, more cheese, more vegetables, more eggs, more everything. And this marvelous increase has been due to the work of scientists of many nations, French, American, German, Russian, English, Danish and many others. Science has transformed agriculture in Western Europe, in Russia, in Canada, in U.S.A., in Australia, and in New Zealand, and is still improving it.

But, I repeat, of the two billions of men, women and children on this earth of ours, a billion and a half are still hungry. No wonder we try not to think of it.

It is not only the horror of it that appals me; it is the stupidity. Let us be thoroughly selfish for the moment. We have enough, probably too much, to eat. Our friends and relatives are probably in the same position, unless they happen to be in England or any part of Europe. So, why worry?

Ask Franklin. Think of his humanity and of his common Sense. Franklin was not ashamed of his humanity, of his sympathy for the poor and hungry. Perhaps this can be best seen in his writings concerning

the Indians and the negroes, though we must not forget his efforts in founding a hospital and many another work of charity.

We, too, self-centred as we are, can't help worrying if we see hunger and want before our eyes. But we can forget hunger and want if we don't actually see it. Now come in both imagination and glorified common sense, such as Franklin's.

Let us go back for a moment to the Indians. Franklin realized that their contribution to the economy of the Colonies was highly important. Without the furs sold by the Indians the whole export and import system would have been imperilled. Towards the end of his career he wrote "It has appeared to me that almost every war between the Indians and whites has been occasioned by some injustice of the latter towards the former. It is indeed extremely imprudent in us to quarrel with them for their lands, as they are generally willing to sell, and sell such good bargains." And he had the courage to speak thus plainly in the days when the frontiersmen saw "no good in an Indian except in a dead one."

Now, let us get back again to the hungry billion and a half human beings, in terms of common sense, Franklin's brand. The world would be a happier and richer place if they had enough to eat. We should be happier and richer if *they* had enough to eat.

No, I am not a Santa Claus. I believe in trade for mutual advantage. Raise the standard of living all over the world. Demand for consumer goods would increase. Producers of consumer goods would benefit accordingly and buy more from the farmer. The ascending spiral!

So simple in theory, so terribly difficult, so slow, in practice. It is for that reason that we must begin with agriculture, the occupation of the large majority of mankind. We want to raise the standard of productivity all over the world. After all everything we have comes from the land or the sea,—a fact that we town dwellers are liable to forget.

Science must be brought to the help of the billion and a half, science applied to agriculture. Science and Education. As I have just said, a difficult and long process. But surely this is the time if ever to make long term plans, when the world is disrupted, when the old channels of trade have silted up, when mankind is on the march.

Agriculture has been disrupted, not so much in this country, but in England and occupied countries. Of the latter it is difficult to speak with any certainty, but England is an interesting example of the changes necessitated by war.

The total cultivated area has declined during the war, owing to military and industrial demands, by over half a million acres. But an enormous increase of output has been obtained, by more intense farming and by ploughing up grassland. Arable acreage has increased in the

United Kingdom since 1939 by one half. The crops have been selected so as to give the greatest food value, notably potatoes. On the other hand, there has been a considerable fall in the number of livestock. More than half of the nation's pigs and a quarter each of its sheep and poultry have vanished. Yet, despite a fall of 38 per cent. in the area under permanent grass, there are actually more cattle than there were before the War. More important still, the number of cows and heifers in milk is slightly higher.

These changes amount to a revolution in British Agriculture, at all events to a reversal of the trends since 1875. But you may ask "why worry?" Why should not British farmers grow what they think fit and proper? How does it affect this country?

The answer is not so simple. Of course, if English farmers produce foods formerly sold to England by American farmers, those American farmers will lose a market. But remember that if English farmers produce more bacon, the Danes may suffer. If English farmers produce more cheese and butter, New Zealand may be in difficulties. And if the Danes and New Zealanders suffer, they will be less able to purchase American radios and refrigerators.

I must apologize for the obviousness of these platitudes. My point is that even England may wish to reorganize her agriculture after the War. Europe certainly will. How much wiser it would be for the nations to discuss these questions immediately and at all events know what the others are planning. They may even be able to come to a mutual agreement. Can we hope that we shall never see again tons of coffee thrown into the sea or burnt to maintain prices? Shades of Franklin! Even the Boston Tea Party was a trial to him. Why, he suggested that payment for the loss should be made—under certain conditions.

Well, Ladies and Gentlemen, I venture to think that I have shown that international cooperation regarding agriculture is necessary today, for the ultimate good of all nations, rich and poor, advanced and less advanced, and I am absolutely convinced that Franklin would have agreed with me had he been alive today.

Had Franklin, the international patriot, been alive to-day, his thoughts would inevitably have centered on the world-embracing problem of today, peace in unity, unity in peace. Has not his major prophecy already been fulfilled?

The United States is now the greatest power in the world, with the greatest army, the greatest navy, the greatest air force, the greatest potential of production. What would Franklin have thought and done under the weight of such a responsibility? We cannot doubt his courage. He would have faced it, without flinching, without boasting.

Does anyone ask "What responsibility"? Why, the responsibility, for America, of world leadership. It is inescapable. Leadership

towards the light, or leadership towards war after war, towards the dark abyss of destruction. There is no middle way. This country must tread one path or the other. That is destiny. To do nothing—is leadership towards the abyss.

What sort of courage is needed today for a nation destined for world leadership? Why Franklin's moral and intellectual courage, the courage he displayed throughout his long life.

As a mere boy taking over the responsibility for a newspaper under suspicion, with the publisher in jail.
As a printer in a foreign town, deceived and friendless.
As the founder of a club for mutual improvement, regardless of laughs and sneers.
As financing small printers in other colonies, ready to run risks for a good cause.
As organizer of a Defence Corps in Pennsylvania, the land of the Quakers.
As a Commissioner calling for justice for the Indians, when massacre was in the air.
As Deputy Postmaster General, facing a serious deficit.
As organizer of transport for General Braddock, the unpopular English general.
As a fearless opponent of the privileged Proprietaries.
As a witness against the Stamp Act before a hostile House of Commons.
As a member of the committee drafting the Declaration of Independence—a challenge to old world tradition.
As a commissioner to obtain aid from France when the horizon loomed blackest.
As a negotiator for peace with Britain when the war fever had not subsided.
As Delegate to the Convention framing a constitution of the United States when agreement seemed impossible.

How Franklin's moral and intellectual courage blazed forth through out the long years! If he felt a course was right morally, he followed it, regardless of public opinion. If he felt that reason impelled him to put forward a point of view, he never hesitated. If circumstances changed he was ready to change his policy. He had the courage even to do that.

Of his toleration and modesty I have already spoken. Let me quote from his own words at that great epoch of his life, the meeting of the Convention to draw up a Constitution for the United States of America.

"Without divine help we shall succeed in this political building no better than the builders of Babel, we shall be divided by our little

partial local interests, our projects will be confounded and we ourselves shall become a reproach and by-word down to future ages. And, what is worse, mankind may hereafter, from this unfortunate instance, despair from establishing governments by human wisdom and leave it to chance war and conquest."

This on June 28th after much wearisome argument. And then on September 17th, when he was pleading for unanimous signatures . . . "I think it will astonish our enemies, who are waiting with confidence that our councils are confounded like those of the builders of Babel and that our States are on the point of separation only to meet hereafter for the purpose of cutting one another's throats."

Ladies and Gentlemen at this solemn moment in the history of the World, what can we do better than to ponder on the words of the great philosopher? What he said in 1787 is doubly true in 1945. Our enemies are gloating at any sign of disunity. They prophesy that we shall be divided by little local partial interests, that we are on the point of separation only to meet hereafter for the purpose of cutting one another's throats. And the rest of the suffering tortured world is praying that we shall manifest our unanimity, and prove that governments can be established by human wisdom, and not left to chance, war and conquest.

Would that Benjamin Franklin were alive today! May his spirit guide us in the tasks that lie before us.

THEORY OF BENDING, TORSION AND BUCKLING OF THIN-WALLED MEMBERS OF OPEN CROSS SECTION.

BY

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INTRODUCTION.

The first problems of instability of thin-walled members were solved about half a century ago, but most of the developments in this field have been accomplished in more recent times, principally in connection with the development of aeroplane structures. The requirements of weight-saving necessitate the wide use of thin-walled members and the analysis of deformations and instability of such structural elements becomes of utmost practical importance. In this discussion we begin with the analysis of bending of thin-walled members of open cross section and establish the notion of *shear center*. Then, by the use of Maxwell's reciprocal theorem we come to torsion and establish the identity of shear center and *center of twist*. After the discussions of bending and torsion, we take up instability problems and discuss torsional buckling and lateral buckling of beams. It is felt that with this order of discussion a considerable simplification in presentation of the theory of thin-walled structures is obtained and certain clarification of the phenomenon of torsional buckling is accomplished.

1. **Pure Bending of Prismatical Bars.**—Let us consider a prismatical bar of arbitrary cross section. We assume that the forces are applied at the ends and are equivalent to two equal and opposite couples acting in an axial plane of the bar. In such a case we have pure bending. The cross sections of the bar remain plane during this bending and only

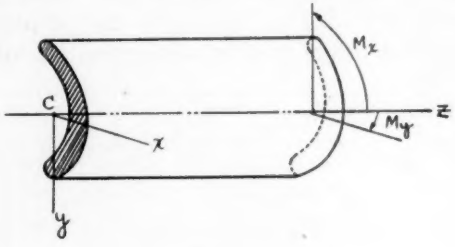


FIG. 1.

rotate with respect to neutral axes which pass through centroids of cross sections. We take the origin of coördinates at the centroid C of one of the cross sections, direct the z-axis along the axis of the bar, the

x-axis along the neutral axis, and select the direction of the y-axis so as to obtain the right-handed coördinate system shown in Fig. 1. Selecting the positive directions of moments M_x, M_y and angles of rotation of cross sections in accordance with the right-hand screw rule as shown in the figure and denoting by r_x the radius of curvature in the yz-plane, taken positive if the angle of rotation of cross sections increases as z increases, we obtain

$$\left.\begin{aligned} \sigma_z &= \frac{Ey}{r_x}, \\ M_x &= \int_A \sigma_z y dA = \frac{EI_x}{r_x}, \\ M_y &= -\int_A \sigma_z x dA = -\frac{EI_{xy}}{r_x}. \end{aligned}\right\} \tag{1}$$

It is seen that the plane in which the bending moment acts coincides with the plane of the elastic line only if $I_{xy} = 0$, i.e., if the bending moment acts in one of the *principal planes* of the bar.

Similar discussion can be repeated for the case in which the neutral axes become parallel to the y-axis, and with the assumed rule of signs we obtain

$$\left.\begin{aligned} \sigma_z &= -\frac{Ex}{r_y}, \\ M_x &= \int_A \sigma_z y dA = -\frac{EI_{xy}}{r_y}, \\ M_y &= -\int_A \sigma_z x dA = \frac{EI_y}{r_y}. \end{aligned}\right\} \tag{2}$$

Having these two particular cases we can, by superposition, investigate bending of the bar by moments acting in any axial plane. Consider, for example, bending by couples acting in the yz-plane. In this case M_y vanishes. To accomplish this by superposition of eqs. (1) and (2) we must have

$$-\frac{EI_{xy}}{r_x} + \frac{EI_y}{r_y} = 0,$$

from which

$$\frac{1}{r_y} = \frac{I_{xy}}{I_y} \cdot \frac{1}{r_x}.$$

The same superposition gives

$$M_x = \frac{EI_x}{r_x} - \frac{EI_{xy}}{r_y} = \frac{E(I_xI_y - I_{xy}^2)}{I_y} \cdot \frac{1}{r_x},$$

and we obtain

$$\left.\begin{aligned}\frac{1}{r_x} &= \frac{M_x I_y}{E(I_x I_y - I_{xy}^2)},\\ \frac{1}{r_y} &= \frac{M_x I_{xy}}{E(I_x I_y - I_{xy}^2)}.\end{aligned}\right\} \tag{3}$$

Having these equations for curvatures in the yz- and xz-planes, we can calculate bending stresses from the equation

$$\sigma_z = \frac{Ey}{r_x} - \frac{Ex}{r_y} = \frac{M_x}{I_x I_y - I_{xy}^2}(I_y y - I_{xy} x). \tag{4}$$

These stresses vanish if the expression in parenthesis is zero. Hence we obtain the equation $I_y y - I_{xy} x = 0$ for the neutral axis. Similarly, if the couples M_y are acting in the xz-plane, we obtain

$$\frac{1}{r_x} = \frac{M_y I_{xy}}{E(I_x I_y - I_{xy}^2)}, \qquad \frac{1}{r_y} = \frac{M_y I_x}{E(I_x I_y - I_{xy}^2)}, \tag{3'}$$

$$\sigma_z = \frac{M_y}{I_x I_y - I_{xy}^2}(I_{xy} y - I_x x). \tag{4'}$$

The results obtained can be rigorously applied only if the forces are distributed over the ends of the bar in the same manner as the stresses in any intermediate cross section. However, the application of the formulas is usually extended to other cases of force distribution by assuming that the replacement of one system of forces by another, statically equivalent, produces only local changes of stresses near the ends and has no substantial effect on the deformation of the bar if its cross-

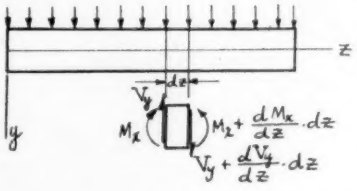


FIG. 2.

sectional dimensions are small in comparison with the length. The limitations of this principle, first formulated by S. Venant, in the case of thin-walled members with open cross section will be discussed later in more detail.

2. **Bending of Prismatical Bars by Transverse Forces.**—Assume that a uniform bar is bent by a system of forces parallel to the y-axis and acting in a plane parallel to the yz-plane, Fig. 1, and at such a distance from that plane that there is only bending of the bar without torsion,

i.e., cross sections of the bar rotate, during deformation, with respect to the x- and y-axes, but do not rotate with respect to the z-axis. Let the magnitudes of bending moment and shearing force at any cross section be M_x and V_y respectively. Then from the consideration of equilibrium of an element of the bar between two adjacent cross sections, Fig. 2, we obtain the following equations:

$$\frac{dM_x}{dz}dz - V_y dz = 0,$$

$$\frac{dV_y}{dz}dz + q_y dz = 0,$$

which give

$$\left.\begin{aligned} \frac{dM_x}{dz} &= V_y, \\ \frac{dV_y}{dz} &= -q_y. \end{aligned}\right\} \tag{5}$$

Here q_y is the intensity of the distributed load. Similarly considering

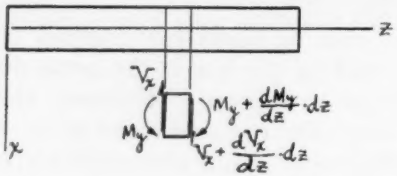


FIG. 3.

transverse loading parallel to the xz-plane, Fig. 3, we obtain

$$\left.\begin{aligned} \frac{dM_y}{dz} &= -V_x, \\ \frac{\partial V_x}{dz} &= -q_x. \end{aligned}\right\} \tag{6}$$

Continuing with the case in Fig. 2, we assume that the bending moment M_x produces bending of the axis, defined by eqs. (3), and bending stresses * given by eq. (4). The shearing forces V_y produce shearing stresses. The calculation of these stresses represents a complicated problem which is rigorously solved only for a few shapes of cross section. If the bar has an axial plane of symmetry and the load is acting in that plane we conclude, from symmetry, that such a load produces only bending in the plane of symmetry, and no torsion. If there are two planes of symmetry, we can resolve any transverse load, passing through

* Rigorous solutions of the theory of elasticity substantiate this assumption.

the axis of the bar, into two components acting in the planes of symmetry and conclude that such a load does not produce torsion.

In the general case we will find that shearing stresses, produced by shearing force V_y, give a resultant which usually does not pass through the centroid of the cross section. This indicates that the load must act in a plane some distance from the axial plane yz in order to produce only bending without torsion. For example, in the case of a cantilever of a semicircular cross section, Fig. 4, to produce bending during which

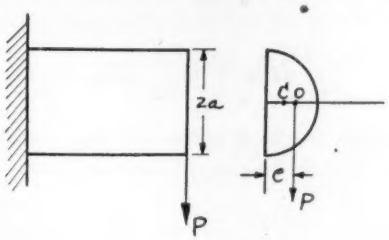


FIG. 4.

the vertical side of the cross section does not rotate the force P must pass through the point O whose distance from the vertical side of the cross section is *

$$e = \frac{8}{15\pi}\frac{3+4\mu}{1+\mu}a,$$

where a is the radius of the cross section and μ is Poisson's ratio for the material. This distance is always larger than the distance of the centroid C from the vertical side of the cross section.

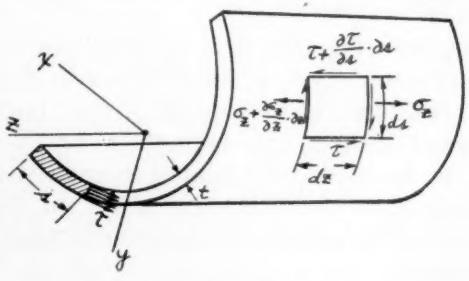


FIG. 5.

The displacement of the plane of action of the transverse load, required to eliminate torsion, becomes of great practical importance in the case of thin-walled members of open cross section, Fig. 5. For this case an approximate solution can be readily obtained.† In the case of

* This question was discussed by the writer. See his paper in *Bull. of the Institute of Engrs of Ways of Communication*, S. Petersburgh, 1913.

† This approximate solution was obtained and its practical significance was explained by R. Maillart, *Schweizerische Bauzeitung*, Vol. 77, p. 195 (1921), and Vol. 79, p. 254 (1922).

thin walls it can be assumed that the shearing stresses τ are uniformly distributed over the thickness t and are parallel to the corresponding tangent to the *middle line* of the cross section. Along the middle they vary with the distance s from the edge,‡ Fig. 5, and can be calculated from the equation of equilibrium of one element of the wall such as is shown in the figure together with positive directions of all stresses. Projecting all forces, acting on the element, on the z-axis and observing that the thickness of the wall t may vary with s, but is independent of z, we obtain the equation

$$\frac{\partial(\tau t)}{\partial s} dsdz + t\frac{\partial \sigma_z}{\partial z} dzds = 0,$$

in which σ_z denotes the normal stress at any point of the middle line of the cross section. From this equation follows

$$\frac{\partial(\tau t)}{\partial s} = -t\frac{\partial \sigma_z}{\partial z}. \tag{7}$$

Substituting for σ_z its expression (4) and using eqs. (5), we obtain

$$\frac{\partial(\tau t)}{\partial s} = -\frac{tV_y}{I_xI_y - I_{xy}^2}(I_y y - I_{xy}x).$$

Integrating this equation and observing that at the edge, $s = 0$, the shearing stress vanishes, since there are no tangential forces acting on the surface of the bar, we find

$$\tau t = -\frac{V_y}{I_xI_y - I_{xy}^2}\left(I_y\int_o^s ytds - I_{xy}\int_o^s xtds\right). \tag{8}$$

This quantity, representing the shearing force per unit length of the middle line of the cross section, is called the *shear flow*. The integrals on the right side represent the moments of that portion of the cross-sectional area shaded in the figure about the x- and y-axes respectively. These moments can be readily calculated in each particular case, and we obtain the value of τt for each value of s. In the particular case, when x and y are the principal axes of the cross section, we have $I_{xy} = 0$, and equation (8) gives

$$\tau t = -\frac{V_y}{I_x}\int_o^s ytds. \tag{9}$$

Take, for example, a channel section of constant thickness t,* Fig. 6. Selecting the coördinate axes as shown in the figure, we calculate the quantity τt, from eq. (9). The result of this calculation is shown in

‡ s is measured positive in accordance with right-hand screw rule.

* The thickness is assumed very small and the cross section is represented by its middle line only.

the figure by shaded areas. It is seen that along the flanges the shear flow varies linearly while along the web it varies parabolically. Shearing stresses in the web give the vertical resultant represented by the corresponding shaded area and equal to

$$\frac{hV_y}{I_x}\left(\frac{bht}{2}+\frac{2}{3}\cdot\frac{h^2t}{8}\right)=V_y.$$

The shearing stresses in the flanges give a couple the moment of which is $V_y b^2h^2t/4I_x$. Dividing this moment by V_y we find the distance

$$d=\frac{b^2h^2t}{4I_x} \tag{10}$$

defining the position of the resultant shearing force V_y statically equivalent to the calculated system of shearing stresses.

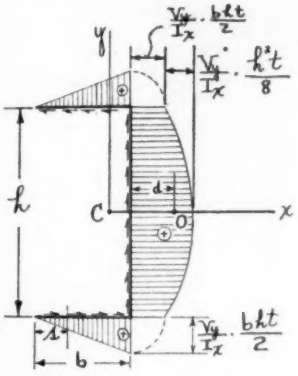


FIG. 6.

It is seen that to produce bending without torsion of a channel by transverse forces parallel to the web the plane of action of these forces must be at the distance d from the middle plane of the web.

Transverse forces perpendicular to the web will not produce torsion if they act in the plane of symmetry xz. The line of intersection of this plane with the previously defined plane of forces parallel to the web is called the *shear center axis*, and its intersection with the xy-plane gives the *shear center* of the cross section. From the above discussion it is clear that any transverse forces distributed along the shear center axis will produce only bending without torsion, since each such force can be resolved into two components acting in the two above defined planes and producing only bending.

As a second example, let us consider a cross section the middle line of which is an arc of a circle, Fig. 7. Again we have one symmetry axis, which we take for the x-axis, and use eq. (9) for calculation shear flow.

Let the thickness of the wall t be constant. Then denoting by a the radius of the middle line and by φ the angle defining the position of a point on the middle line, we have

$$ds = ad\varphi, \qquad y = -a \sin(\alpha - \varphi).$$

Substituting into eq. (9), we obtain

$$\tau t = \frac{V_y}{I_x} a^2 t \int_0^{\varphi} \sin(\alpha - \varphi) d\varphi = \frac{V_y}{I_x} a^2 t [\cos(\alpha - \varphi) - \cos\alpha].$$

We see that the shear flow vanishes for $\varphi = 0$ and $\varphi = 2\alpha$. For $\varphi = \alpha$ it has its maximum value:

$$(\tau t)_{\max} = \frac{V_y}{I_x} a^2 t(1 - \cos\alpha).$$

From symmetry we conclude that the shear center is on the x-axis.

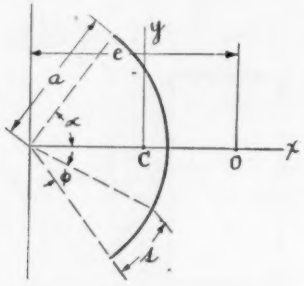


FIG. 7.

Its distance e from the center of the circle is obtained by dividing by V_y the moment of the calculated shearing stresses with respect to the center of the circle. Hence

$$e = \frac{2a}{V_y} \int_0^{\alpha} \tau t ds = 2a \frac{\sin\alpha - \alpha\cos\alpha}{\alpha - \sin\alpha\cos\alpha}. \qquad (11)$$

In the particular case where $\alpha = \pi$, i.e., the bar is a thin circular tube slotted longitudinally, we obtain $e = 2a$. The shear center is outside of the tube in the direction opposite to the slot and at the distance a from the middle surface of the tube.

In the general case when the centroidal axes x and y are not principal axes, Fig. 8, and the load which produces only bending is acting in the plane parallel to the yz-plane, we use eq. (8) for calculating the shear flow. To calculate the distance x_0 of the plane of loading from the centroid C, we calculate the moment with respect to C of the shearing stresses defined by eq. (8) and divide it by V_y. The required distance

then is

$$x_o = \frac{1}{V_y}\int_o^m \tau t r ds = \frac{1}{I_x I_y - I_{xy}^2}\Big[- I_y \int_o^m r ds \int_o^s y t ds + I_{xy} \int_o^m r ds \int_o^s x t ds \Big]. \quad (12)$$

Here m denotes the total length of the middle line of the cross section and r is the distance from the centroid C to a tangent at any point of the middle line of the cross section, as shown in Fig. 8. It is taken positive if a vector coinciding with the tangent and having the direction in which s is increasing, gives a positive moment with respect to C.

A similar analysis can be made if the forces are acting in a plane parallel to the xz-plane. The normal stresses in this case are given by eq. (4′). Substituting it into eq. (7), the following expression for shear

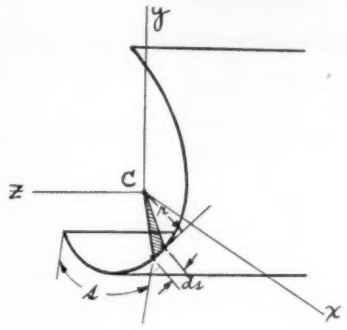


FIG. 8.

flow is obtained

$$\tau t = \frac{V_x}{I_x I_y - I_{xy}^2}\left(I_{xy} \int_o^s y t ds - I_x \int_o^s x t ds \right). \quad (13)$$

Taking the moment of these stresses with respect to the centroid and observing that to positive value of this moment corresponds the displacement of the plane of loading in the direction of negative y, we obtain

$$y_o = - \frac{1}{V_x}\int_o^m \tau t r ds = \frac{1}{I_x I_y - I_{xy}^2}\Big[I_x \int_o^m r ds \int_o^s x t ds - I_{xy} \int_o^m r ds \int_o^s y t ds \Big]. \quad (14)$$

Equations (12) and (14) give us the coördinates of the shear center in the general case.

These equations can be written in simpler form if we observe that rds represents the doubled area of an elemental triangle, shaded in

Fig. 8, and introduce the notation

$$\int_o^s rds = \omega_s$$

for the double sectorial area swept by the radius vector when the distance along the middle line increases from $s = o$ to s. With this notation and integrating by parts we obtain for the first term in the brackets of eq. (12)

$$\int_o^m rds \int_o^s ytds = \int_o^m d\omega_s \int_o^s ytds = \left|\omega_s \int_o^s ytds\right|_o^m - \int_o^m \omega_s ytds.$$

Observing that the integral in the first term on the right-hand side vanishes for both limits, we obtain

$$\int_o^m rds \int_o^s ytds = -\int_o^m \omega_s ytds.$$

Similarly, we can transform the other terms in the brackets of eqs. (12)

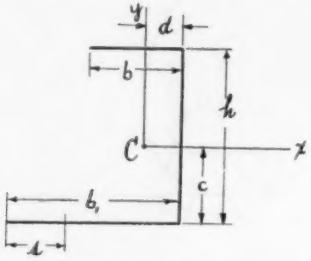


FIG. 9.

and (14) and obtain

$$x_o = \frac{1}{I_x I_y - I_{xy}^2}\left(I_y \int_o^m \omega_s ytds - I_{xy} \int_o^m \omega_s xtds\right), \qquad (12')$$

$$y_o = \frac{1}{I_x I_y - I_{xy}^2}\left(-I_x \int_o^m \omega_s xtds + I_{xy} \int_o^m \omega_s ytds\right). \qquad (14')$$

When x and y are principal axes of the cross section, $I_{xy} = 0$, and we obtain

$$x_o = \frac{1}{I_x}\int_o^m \omega_s ytds,$$

$$y_o = -\frac{1}{I_y}\int_o^m \omega_s xtds.$$

If the middle line of the cross section consists of several straight portions the calculation of the coördinates of the shear center can often be greatly simplified. Take, for example, a channel of uniform thick-

ness but with unequal flanges, Fig. 9. If the forces are acting in a plane parallel to the yz-plane, the resultant of shearing stresses acting on the web is equal to V_y and the horizontal shearing stresses in the flanges give a couple. Considering the lower flange and using eq. (8) we find, by substituting $y = -c$, $x = -(b_1 - d - s)$,

$$\tau t = \frac{V_y}{I_x I_y - I_{xy}^2}\left[I_y cts + I_{xy} t \left(\frac{s^2}{2} + ds - b_1 s \right) \right].$$

The resultant shearing force in the flange is

$$\int_o^{b_1} \tau t ds = \frac{V_y b_1^2 t}{I_x I_y - I_{xy}^2}\left[I_y \frac{c}{2} + I_{xy} \left(\frac{d}{2} - \frac{b_1}{3} \right) \right].$$

Multiplying it by h and adding the moment $V_y d$ of shearing stresses in the web, we obtain the centroidal moment of all shearing stresses distributed over the cross section. Dividing this moment by V_y, we obtain

$$x_o = \frac{b_1^2 ht}{I_x I_y - I_{xy}^2}\left[I_y \frac{c}{2} + I_{xy} \left(\frac{d}{2} - \frac{b_1}{3} \right) \right] + d.$$

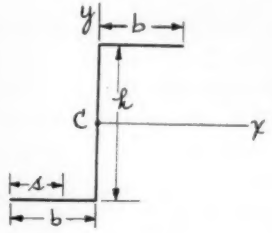


FIG. 10

To calculate the coördinate y_o we assume the load in a plane parallel to the xz-plane. The total shear force V_x is divided between the two flanges. The shear flow for the lower flange is obtained from eq. (13), which gives

$$\tau t = \frac{V_x}{I_x I_y - I_{xy}^2}\left[- I_{xy} cts + I_x t \left(b_1 s - ds - \frac{s^2}{2} \right) \right].$$

The shearing force in the lower flange is

$$\int_o^{b_1} \tau t ds = \frac{V_x b_1^2 t}{I_x I_y - I_{xy}^2}\left[- I_{xy} \frac{c}{2} + I_x \left(\frac{b_1}{3} - \frac{d}{2} \right) \right].$$

Multiplying this force with h and dividing by V_x, we obtain the distance of the resultant shearing force V_x from the upper flange and the second coördinate of the shear center is

$$y_o = h - c - \frac{b_1^2 t}{I_x I_y - I_{xy}^2}\left[- I_{xy} \frac{c}{2} + I_x \left(\frac{b_1}{3} - \frac{d}{2} \right) \right].$$

In the case of a z-section considering the load parallel to the yz-plane and using eq. (8), we find that the shearing force in the lower flange vanishes. It must vanish also in the upper flange, hence $x_o = o$. Considering now the load parallel to the xz-plane and using eq. (13), we find that the shearing force in the flange is equal to $V_x/2$, which indicates that the plane of loading must contain the centroid C. Hence $y_o = o$, and the shear center coincides with the centroid in this case.

In the cases where all flanges of a bar intersect along one axis, as shown in Fig. 11, the shearing force also passes through the same axis and this axis is evidently the shear center axis.

For calculating deflections produced in a bar by any transverse load distributed along the shear center axis, we resolve that load into two components parallel to the x- and y-axes and calculate the bending

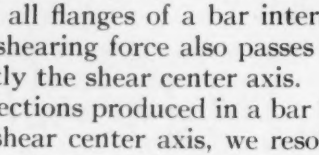


FIG. 11.

moments, M_x and M_y, in the yz- and xz-planes, respectively. Then, by using eqs. (3) and (3′), we obtain the following expressions for curvatures:

$$\left.\begin{aligned} \frac{1}{r_x} &= \frac{1}{E(I_xI_y - I_{xy}^2)}(M_xI_y + M_yI_{xy}), \\ \frac{1}{r_y} &= \frac{1}{E(I_xI_y - I_{xy}^2)}(M_xI_{xy} + M_yI_x). \end{aligned}\right\} \tag{15}$$

If x and y are principal axes, these equations reduce to

$$\frac{1}{r_x} = \frac{M_x}{EI_x}, \qquad \frac{1}{r_y} = \frac{M_y}{EI_y}. \tag{16}$$

Let us denote the deflections in the x and y directions by u and v. Then, using the sign rule established in article 1 we obtain for small deflections

$$\frac{1}{r_x} = -\frac{d^2v}{dz^2}, \qquad \frac{1}{r_y} = \frac{d^2u}{dz^2},$$

and eqs. (16) give

$$\left.\begin{aligned} EI_x\frac{d^2v}{dz^2} &= -M_x, \\ EI_y\frac{d^2u}{dz^2} &= M_y. \end{aligned}\right\} \tag{17}$$

Differentiating these equations twice with respect to z and using eqs. (5) and (6), we obtain

$$\left.\begin{aligned} EI_x \frac{d^4v}{dz^4} &= q_y, \\ EI_y \frac{d^4u}{dz^4} &= q_x. \end{aligned}\right\} \tag{18}$$

If the transverse forces do not pass through the *shear center axis* they produce not only bending but also torsion of the bar. In discussing such problems, we replace each force which does not pass through the shear center axis by a parallel force passing through that axis and a couple acting in a plane perpendicular to the axis of the bar. In this way the problem is divided into two parts: (1) the investigation of bending produced by forces distributed along the shear center axis and (2) the investigation of torsion by couples acting in the planes perpendicular to the axis of the bar. The first part of the problem has already been discussed. The part dealing with torsion will be discussed in the next article.

3. **Pure Torsion of Thin-Walled Members of Open Cross Section.**—If a bar is twisted by couples applied at the ends in the planes perpendicular to the axis of the bar, and the ends are free to warp, we have the case of *pure torsion*. There will be produced in this case only shearing stresses on each section of the bar. The law of distribution of these stresses depends on the form of the cross section and is the same for all cross sections. In the case of thin-walled open sections it can be assumed with sufficient accuracy that shearing stress at any point is parallel to the corresponding tangent to the middle line of the cross section and is proportional to the distance from that line. The angle of twist per unit length is

$$\theta = \frac{T}{C}, \tag{19}$$

where T denotes the torque and C is the *torsional rigidity* of the bar. In the case of a thin-walled open section of constant thickness t, we can take with sufficient accuracy

$$C = \tfrac{1}{3} m t^3 G, \tag{20}$$

where m is the length of the middle line of the cross section and G the modulus of elasticity in shear. If the cross section consists of several portions of different thickness we can assume

$$C = \tfrac{1}{3} \sum m_i t_i^3 G. \tag{21}$$

The longitudinal fibers of the bar are deformed during twist to helixes, which, for small angle of twist, can be considered as straight

lines inclined to the axis of rotation. If ρ denotes the distance of the fiber from the axis of rotation the angle of inclination of the fiber to the axis is $\rho\theta$.

Warping of the cross section in the case of a thin-walled open section can be easily visualized if we observe that there is no shearing stress

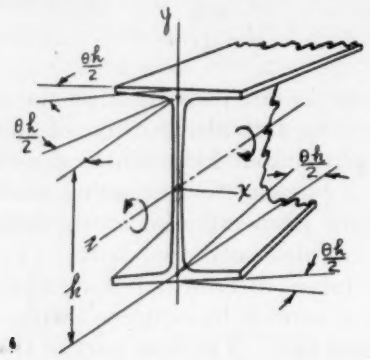


FIG. 12.

along the middle line of the cross section, which indicates that the elements of this line remain normal to longitudinal fibers which become helixes after torsion. In Fig. 12, for example, the warping of the cross section of a twisted I-beam is shown. During rotation with respect to the centroidal axis of the beam, the central fibers of the flanges, distant

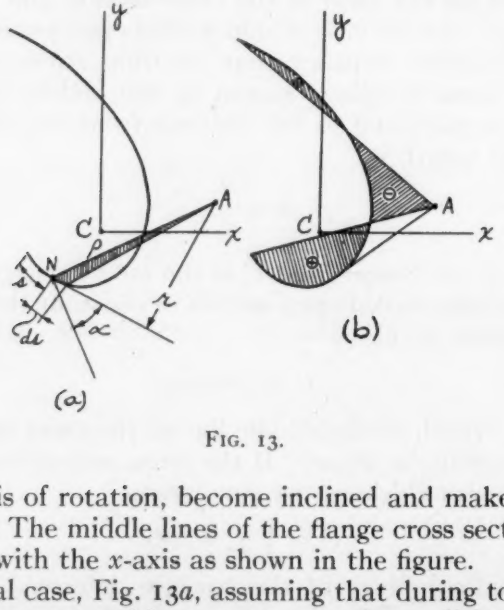


FIG. 13.

$h/2$ from the axis of rotation, become inclined and make with this axis the angle $\theta h/2$. The middle lines of the flange cross sections will make the same angle with the x-axis as shown in the figure.

In the general case, Fig. 13a, assuming that during torsion the cross sections of the bar rotate with respect to an axis parallel to the centroidal

axis and passing through point A, any longitudinal fiber N in the middle surface of the wall, defined by the distance s measured along the middle line of the cross section, becomes inclined to the axis of rotation by the angle $\rho\theta$. The element of the tangent to the middle line at N remains perpendicular to the fiber and makes with the xy-plane, after torsion, a small angle $\rho\theta \cos \alpha = r\theta$, where r is the distance of the tangent from the axis of rotation, taken positive if a vector along the tangent and pointing in the direction of increasing s gives a positive moment with respect to the axis of rotation. Let w denote displacements in the z-direction, of points of the middle line of the cross section. Then, for positive torque, we have

$$\frac{\partial w}{\partial s} = -r\theta. \tag{22}$$

By integration, we obtain

$$w = w_o - \theta \int_o^s r ds, \tag{23}$$

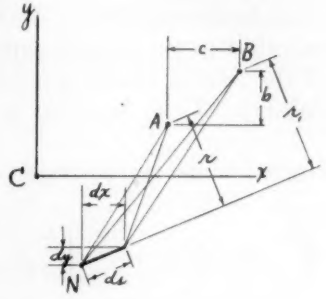


FIG. 14.

where w_o denotes the displacement in the z-direction of the point from which s is measured. The integral on the right-hand side represents, as we have already seen, the doubled sectorial area swept by the radius ρ as we move along the middle line of the cross section from the edge, $s = o$, up to the point N under consideration. The swept area is taken positive when the radius ρ is rotating in the positive direction, i.e., from x to y, and negative for rotation in the opposite direction. The value of the integral for $s = m$ will then be represented by the double algebraic sum of the three areas shaded in Fig. 13b.

In the previous discussion we assumed rotation of the cross section with respect to an arbitrarily chosen point A. Let us consider now the effect on warping of displacement of the center of rotation. Assume, for example, that the center of rotation is moved from A to B, Fig. 14. Considering one element ds of the middle line and denoting by x, y the coördinates of point N and by x_a, y_a the coördinates of the center of

rotation A, we see from the figure that

$$rds = dx(y_a - y) - dy(x_a - x).$$

If we move the center of rotation from A to B we have to take $x_a + c$ and $y_a + b$ for the coördinates of the new center of rotation. Then

$$r_1 ds = rds + bdx - cdy.$$

To calculate warping produced by rotation about B, we have to substitute $r_1 ds$, instead of rds, into eq. (23) and we obtain

$$\int_o^s r_1 ds = \int_o^s rds + \int_o^s (bdx - cdy) = \int_o^s rds + bx - cy + \text{const.} \quad (24)$$

It is seen that the change in position of the center of rotation results in addition to the previously calculated displacements (23) of the displacement

$$bx - cy + \text{const.},$$

which, as a linear function of x and y, does not require any additional deformation of the bar and is accomplished by only moving the bar as a rigid body. This shows that in the case of pure torsion of a bar with free ends the selection of the axis of rotation is immaterial and any line parallel to the centroidal axis of the bar can be taken as the axis of rotation.

Using eq. (23) we can calculate the average value w_a of warping as follows:

$$w_a = \frac{1}{m}\int_o^m wds = w_o - \frac{\theta}{m}\int_o^m ds \int_o^s rds. \quad (25)$$

Subtracting this value from the previously calculated displacement (23), we obtain the warping of the cross section with respect to the plane of average w. Continuing to use the symbol w for displacements with respect to the new reference plane, we obtain

$$w = \theta\left(\frac{1}{m}\int_o^m ds \int_o^s rds - \int_o^s rds\right). \quad (26)$$

The first term in parenthesis is a constant depending on geometrical dimensions of the middle line of the cross section, while the second term represents the double sectorial area corresponding to the arc s of the middle line of the cross section. To simplify writing, we introduce the notations

$$\left.\begin{aligned} \frac{1}{m}\int_o^m ds \int_o^s rds &= D, \\ \int_o^s rds &= \omega_s. \end{aligned}\right\} \quad (27)$$

Then

$$w = \theta(D - \omega_s). \tag{28}$$

Let us apply this equation to calculations of warping in several simple cases. As a first example, we take the circular arc, Fig. 15, with

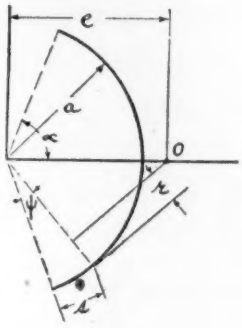


FIG. 15.

an angle 2α. Taking the shear-center O as the center of rotation, we have

$$r = a - e\cos(\alpha - \psi), \qquad ds = ad\psi,$$

$$\int_o^s rds = \omega_s = a\int_o^\psi [a - e\cos(\alpha - \psi)]ds = a[a\psi + e\sin(\alpha - \psi)],$$

$$\frac{1}{m}\int_o^m ds\int_o^s rds = D = \frac{a}{2\alpha}\int_o^{2\alpha}[a\psi + e\sin(\alpha - \psi)]d\psi = a^2\alpha,$$

and warping, as represented by eq. (28), is

$$w = \theta a[a(\alpha - \psi) - e\sin(\alpha - \psi)].$$

We see that warping is antisymmetrical with respect to the axis of symmetry of the cross section and vanishes at $\psi = \alpha$.

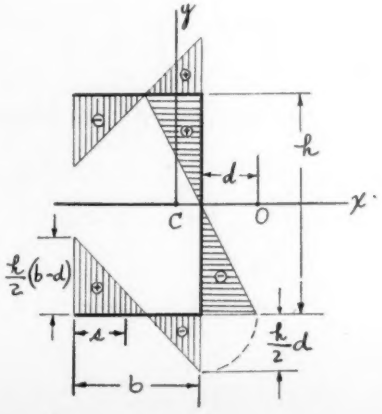


FIG. 16.

As a second example, let us consider the case of a channel, Fig. 16. Taking rotation with respect to the shear center O, we find the following expressions for the doubled sectorial areas of the three portions of the cross section:

$$\omega_s = \int_o^s r\,ds = \frac{h}{2}\,s, \qquad \text{for} \qquad s < b,$$

$$\omega_s = \int_o^s r\,ds = \frac{h}{2}\,b - d(s-b), \qquad \text{for} \qquad b < s < b+h,$$

$$\omega_s = \int_o^s r\,ds = \frac{h}{2}\,b - dh + \frac{h}{2}\,(s-h-b), \qquad \text{for} \qquad s > b+h.$$

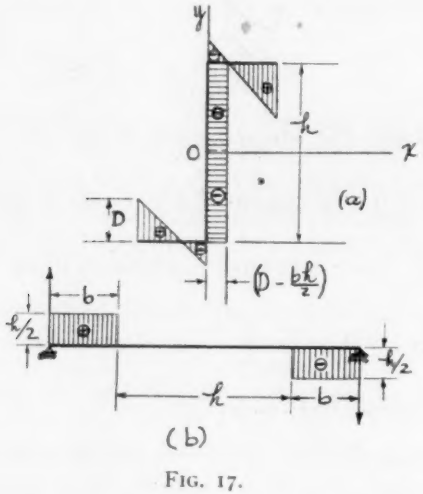


FIG. 17.

Using these expressions, we obtain

$$D = \frac{1}{2b+h}\int_o^{2b+h} ds \int_o^s r\,ds = \frac{h(b-d)}{2}.$$

Substituting into eq. (28), we obtain the following expressions for warping:

$$w = \theta\,\frac{h}{2}\,(b-d-s), \qquad \text{for} \qquad s < b,$$

$$w = \theta\left(-\frac{dh}{2} - bd + ds\right), \qquad \text{for} \qquad b < s < b+h,$$

$$w = \theta\,\frac{h}{2}\,(d+h+b-s), \qquad \text{for} \qquad s > b+h.$$

The variation of w along the middle line of the cross section for $\theta = 1$ is shown by shaded areas in Fig. 16.

In calculations of warping, a beam analogy can be used to advantage. Imagine the middle line of the cross section straightened and consider it as a beam of length m loaded by fictitious load of intensity r. In such a case ω_s represents the summation of the load at the distance s from the end, $\int_o^s ds\omega_s$ is the moment of that load for the same point and the quantity D represents the reaction at the support of the beam. Thus the quantity $D - \omega_s$ is the shearing force in the beam. Take, for example, the section shown in Fig. 17*a*. Considering rotation about the shear center O, we find $r = \frac{h}{2}$ for the lower flange, $r = o$ for the web and $r = -\frac{h}{2}$ for the upper flange. The corresponding fictitious load is as shown in Fig. 17*b*. The reaction at the left end is

$$D = \frac{h}{2} b \frac{(h + b)}{(h + 2b)} .$$

The values of the shearing force are indicated by shaded areas in Fig. 17*a*. Multiplied with θ they give the warping of the cross section in this case.

In the case of cross sections all flanges of which are intersecting in one point, Fig. 11, r vanishes for all points of the middle line and there will be no warping of this line during torsion.

(To be continued in April issue.)

Quick Freezing Makes Meat Tender.—Formation of ice crystals within the fibers of meat that is frozen, resulting in splitting of the fibers, is believed to be the explanation of the "tenderizing" effect of sharp freezing of meat. At temperatures only a few degrees below the freezing point, U. S. Department of Agriculture scientists did not find ice crystals within the fibers when they were investigating the effects of low temperature on the tenderness of beef. At zero Fahrenheit, crystals were evident, and at 10 below and 40 below the crystals were finer, there were more of them, and the frozen meat was more tender when thawed and cooked.

The experimenters, O. G. Hankins and R. L. Hiner of the Bureau of Animal Industry, report that there has long been a prejudice against frozen meat. These recent laboratory measurements of the actual effects on tenderness should help greatly, they believe, in leading consumers to appreciate the effects of good methods of freezing.

In general, aging or "ripening" has been depended on to make meat more tender. After beef or lamb is slaughtered, it is often kept hanging in a cooler for as long as several weeks, at a temperature only slightly above freezing. This prevents rapid multiplication of the bacteria that cause spoilage, but does not prevent the action of enzymes that are already in the meat. The action of the enzymes is in some respects similar to digestion, and the effect on the muscle fibers and connective tissue is to make them more tender.

In a study of aging for as long as 35 days and freezing at 20 above and 10 below zero, the best combination to keep deterioration down and obtain a good tenderizing effect was to age the meat for 15 days and then freeze it at 10 degrees below zero F. Beef aged only 5 days and frozen at 10 below zero was as tender as beef aged 35 days without freezing.

R. H. O.

FORMALDEHYDE-HYDROQUINONE DEVELOPERS AND INFECTIOUS DEVELOPMENT.

BY

J. A. C. YULE.

Communication No. 989 from the Kodak Research Laboratories.

INTRODUCTION.

It is well known that by the use of fine-grain films, together with formaldehyde-hydroquinone developers, such as the Kodak D-85,* unusual results can be obtained. These include a very high gamma, but the most striking and valuable feature is the fact that the characteristic curve has an extremely short toe (see Fig. 1), which results in very clean and sharp line and halftone negatives. This high contrast is unfortunately accompanied by a tendency for fine clear lines to be filled in and for black lines to be broadened.

The present paper shows that the explanation of this singular behavior lies in a phenomenon known as "infectious development."

In general, development of an exposed grain of a photographic emulsion does not cause the development of neighboring unexposed grains.[1] An exception to this is the formation of clumps of grains in some emulsions, in which the exposure of one grain causes development of the whole clump.[2] But apart from this clumping, the existence of infectious development, as it has been called, has never been definitely proved. It is not easy to detect a small amount of infectious development, because of the difficulty of exposing certain grains without affecting others. The existence of some type of image spreading with formaldehyde-hydroquinone developer has, however, been suspected for many years, because of the tendency of fine lines to veil over in line

* Water, about 90° F. (30° C.)	500 cc.
Kodak Sodium Sulfite, desiccated	30.0 grams
Kodak Paraformaldehyde	7.5 grams
Kodak Potassium Metabisulfite	2.6 grams
Kodak Boric Acid, crystals	7.5 grams
Kodak Hydroquinone	22.5 grams
Kodak Potassium Bromide	1.6 grams
Water to make	1.0 liter

[1] Mees, C. E. K., "The Theory of the Photographic Process," The Macmillan Company, New York, 1942, p. 832.

Svedberg, T., "The Reducibility of the Individual Halide Grain in a Photographic Emulsion," *Phot. J.*, **62**: 183, 1922.

[2] Trivelli, A. P. H., Righter, F. L. and Sheppard, S. E., "On the Mutual Infection of Contiguous Silver Halide in Photographic Emulsions," *Phot. J.*, 62: 407, 1922.

negatives developed in this type of developers, and the experiments to be described leave little doubt as to its presence.

The Eberhard effect and the Mackie line depend upon the lowered activity of the exhausted developer due to the accumulation of soluble halides, the lowering of pH, and the depletion of the developing agent. In addition to these changes, developer oxidation products are formed, which Luther and Frötschner [3] found to be capable of greatly increasing the activity of hydroquinone developers. It might be expected, therefore, that a partially exhausted developer would often be more active

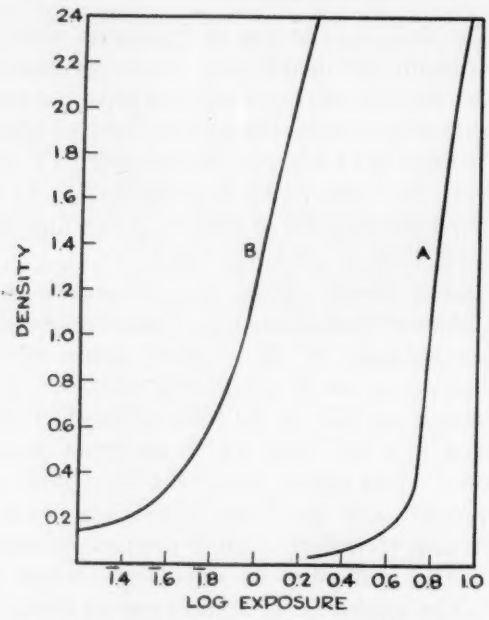


FIG. 1. Kodalith film developed in Kodalith developer (Curve A) and in D-8 (Curve B) for 1½ minutes at 68° F.

than a fresh developer, instead of less active. If this were the case, the effect produced would be exactly the opposite of the Eberhard effect and the Mackie line, that is, large images would spread beyond the proper boundaries and small images would be lower in density than the large ones. Grains would develop more rapidly if they happened to be in the neighborhood of other developing grains; in other words, infectious development would occur. The fact that this has not previously been proved is probably due to the short life of the oxidation products, which, in normal developers, are rapidly removed by reaction with sulfites.

[3] Frötschner, H., "Zur Arbeitsweise zusammengesetzter Entwickler," *Phot. Ind.*, 35: 801, 1937.

In order to study infectious development, or any development effect due to the proximity of developing grains, such as the Eberhard effect, it must be possible to expose some grains without affecting others, and to differentiate between them after development. This is most easily done by studying the edge of an image. The existence of infectious development was first proved by studying the effects at the edge of a halftone dot.

However, it was desirable to check these results in other ways. The necessary selective exposure may also be achieved by the use of differently color-sensitized emulsions, either mixed or coated on top of each other; by the use of mixed or double-coated emulsions of widely differing speeds (though this is not so applicable to Kodalith); or by mixing or double-coating emulsions, one of which is fogged.

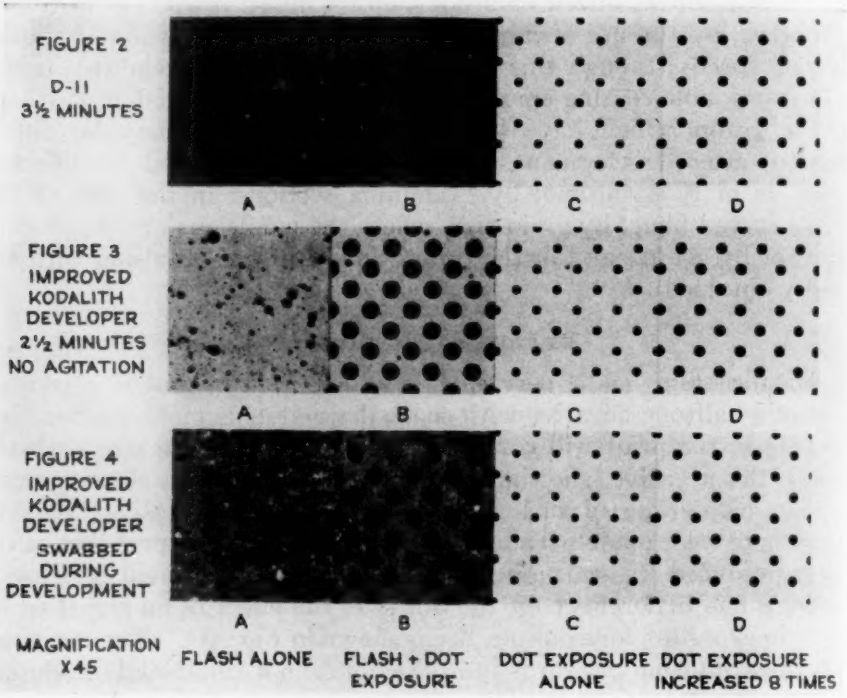
The grains which have been selectively exposed may be differentiated, after development, by the color, if developed to different colors, as in Kodachrome; by examining sections, in the case of the double-coated films; by grain size, where the emulsions are of different types; or by comparison of the proportion of fogged emulsion with the density obtained.

HALFTONE-DOT EXPERIMENTS.

Kodalith film has a very high resolving power, and if a contact print of a halftone negative with sharp dense dots is made on this film, the dots in the print will correspond very accurately in size with the dots in the negative, showing no evidence of infectious development. A photomicrograph of such a print is shown in Fig. 3C. The dots shown here were made with ultraviolet light to reduce spreading of the image produced by scattering of light. Moreover, a great increase in exposure has little effect on the dot size, the effect of an eightfold increase in exposure, for example, being shown in Fig. 3D. The activated developer diffusing outwards from the dots is not capable of developing the unexposed parts of the film. If, however, the film is given a very short over-all exposure in addition to the dot exposure, the result is quite different. The over-all exposure may be sufficient to produce only a slight density, as in Fig. 3A.* The dots are greatly increased in size as in Fig. 3B. In fact, the increase in size is very much greater than that caused by increasing the dot exposure eight times. This may be explained by the hypothesis that, while the activated developer diffusing outwards from the dots is not capable of developing unexposed grains, it is capable of greatly accelerating the development of the slightly exposed grains of the background. In this way, it differs from

* The black spots produced by the uniform exposure have been called "pepper" spots, and are also due to infectious development spreading from grains or clumps of grains which for some reason begin to develop a little earlier than the others.

the infectious development produced by traces of hydrazine described by Stauffer, Trivelli, and Smith.[4] Further evidence that the high density around the dots is not connected with the scattering of light around the dot is shown by developing the film in D-11 instead of Kodalith developer, as in Fig. 2. The addition of a flash exposure, *A*, to the dot exposure, *C*, does not appreciably increase the dot size with this developer, as it would if scattering of light were responsible for the image spreading.



FIGS. 2, 3 AND 4. Photomicrographs showing image spreading with Kodalith developer.

In the experiments of Fig. 3, the film was not agitated during development, in order to enhance the effect. However, as in Fig. 4, infectious development occurs to some extent even with the greatest possible agitation, produced by swabbing the film constantly during development.

As will be seen in Fig. 5, infectious development can take place on the same film as the Mackie line; a dot exposure alone (Fig. 5C) combined with an over-all exposure (Fig. 5A) produces an increase in dot

[4] Stauffer, R. E., Trivelli, A. P. H., and Smith, W. F., J. FRANKLIN INSTITUTE, 238: 291, October, 1944.

size (image spreading) while greatly reducing the background density caused by the over-all exposure. This reduction in density would have shown itself as a Mackie line if the dots had been farther apart. The enhanced activity is apparently very short-lived; by the time the developer has diffused 0.1 mm. from the developing area, it has lost its extra activity and behaves like any other exhausted developer.

DOUBLE-LAYER EMULSIONS.

The existence of infectious development was confirmed by experiments on double-layer Kodalith emulsions. The top layer was sensitized to green light, and the lower layer to red light. Nonwandering

FIGURE 5

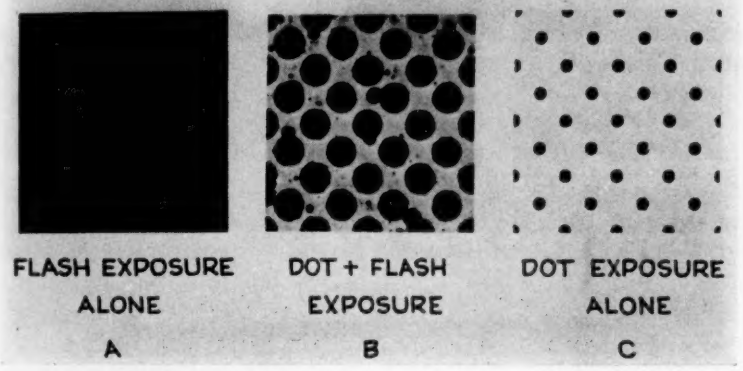


FIG. 5. The enlarged dots in (b) are due to infectious development, while the Mackie line shows itself as a reduction of background density.

sensitizers were used, and the sensitivity of the green layer to red, and the red layer to green, was low enough so that each layer could be exposed separately without affecting the other.

As in the case of the dot-spreading experiments, it was found that the occurrence of development accelerated the development of neighboring regions. In the dot tests previously described, this spreading of the image took place laterally. With the double-layer emulsions, on the other hand, the image spreads from one layer to the other.

When a halftone-dot exposure alone was made with red light, photomicrographs of sections of the film (Fig. 6B) showed very little effect on the green-sensitive layer. When, however, the top layer was given a short uniform exposure with green light, it was found that this layer developed to a much higher density where it was in contact with dots of the red-sensitive layer (Fig. 6A). In a normal type of developer,

however, such as D-11, no effect of this kind was observed, the density produced by the uniform green exposure being uniform, irrespective of the presence of dots in the other layer.

MIXED EMULSIONS.

A Kodalith emulsion was divided into two parts before coating. One part was thoroughly fogged by exposure to light, and a coating was made from a mixture of one part of this with nine parts of the unfogged emulsion. When this was developed fully (6 minutes) in Kodalith developer, a density of 0.56 was produced. This is 12 per

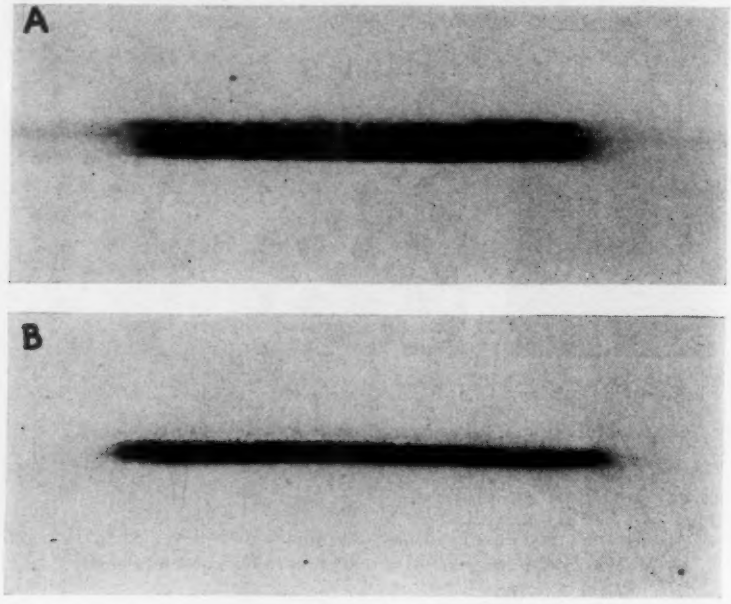


FIG. 6. Photomicrographs showing infectious development in the upper emulsion layer due to an exposure in the lower layer.

cent. of the maximum density which could be produced by exposing the coating further before development. That is to say, the density is only very slightly higher than the theoretical, indicating that practically no infectious development of unexposed grains took place. Ferrous oxalate gave a similar result, but conventional developers gave somewhat higher densities, which was attributed mainly to physical development.

A study of the rate of development of these emulsions gave some interesting results. The rate of development was arbitrarily defined as the time required to produce one half of the final density. With ferrous oxalate developer, the 10 per cent. fogged emulsion developed at exactly

the same rate as the completely fogged coating (see Table I). With Kodalith developer, especially when a special restrainer solution was added to it, the completely fogged coating developed much more rapidly than the 10 per cent. fogged coating, indicating that the developer is more active in the neighborhood of a high concentration of developing grains. Addition of extra sulfite, which destroys the particular properties of Kodalith developer, greatly reduced the difference in the rate of development.

TABLE I.

Developer	Time required for half development (in seconds)		Ratio of development rates
	10% fogged emulsion	100% fogged emulsion	
Kodalith developer	100	55	1.82:1
Kodalith developer with restrainer	350	120	2.92:1
Kodalith developer + 10 g./l. Na_2SO_3	125	100	1.25:1
Ferrous oxalate	190	190	1:1

CONDITIONS NECESSARY FOR INFECTIOUS DEVELOPMENT.

While it is possible that infectious development takes place to some extent in almost all photographic development, it is only under very special conditions that it is strong enough to be noticed. It is most easily observed with emulsions like Kodalith.

It is commonly stated that the object of formaldehyde in these developers is to produce alkali, according to the equation: $H_2O + CH_2O + Na_2SO_3 = CH_2O.NaHSO_3 + NaOH$. It is true that the addition of formaldehyde to sodium sulfite does produce alkali, and the formaldehyde itself or its bisulfite compound may have some specific action; but it appears to be the removal of free sulfite rather than the production of alkali which tends to produce infectious development. Any form of alkali may be used provided it gives the correct pH value and sufficient buffer action.

Figure 7 shows the effect on the characteristic curve of varying the sulfite concentration in a hydroquinone developer which contains no formaldehyde. It is seen that at a concentration of 1 gram per liter the extremely high contrast ($\gamma = 9$) and the absence of toe appear which are characteristic of Kodalith developers. Image spreading was also found to occur to a marked degree at this sulfite concentration.

A study of the equilibrium between formaldehyde and sulfites shows that in Kodalith developer the concentration of free sulfite ion corresponds to approximately 1 per cent. of sodium sulfite. In varying the composition of the developer in different ways, it was found that the action of the developer was almost independent of the concentration

of the formaldehyde-bisulfite compound, free formaldehyde, or total sulfite. In order to obtain the effect, however, the concentration of sulfite ion must correspond to about 1 per cent. of sodium sulfite. This concentration is not maintained and the sulfite is readily exhausted unless formaldehyde is present. It seems probable that the main function of the formaldehyde is to act as a "sulfite buffer," which liberates more sulfite as the latter is used up. Formaldehyde is, however, not a very efficient sulfite buffer at this pH value, and a larger

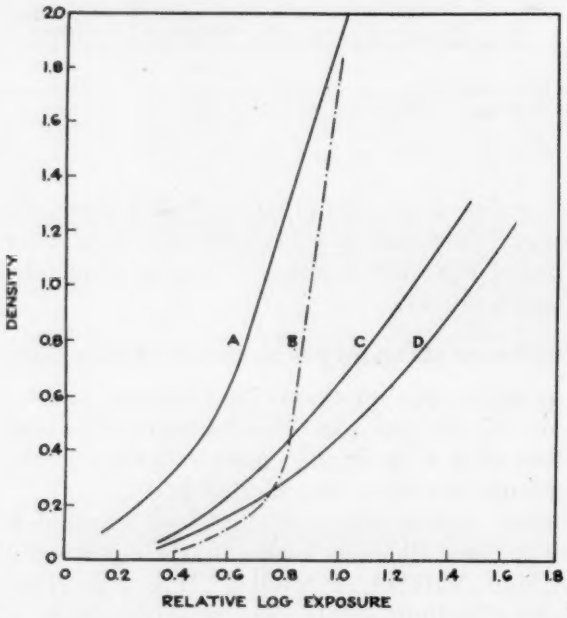


FIG. 7. Effect of varying sulfite concentration in a hydroquinone developer. Composition: Hydroquinone, 15 g/l.; potassium bromide, 1 g/l.; sodium carbonate, 30 g/l.; sodium bicarbonate, 40 g/l.; sodium sulfite content: Curve *A*, none; Curve *B*, 1 g/l.; Curve *C*, 4 g/l.; Curve *D*, 40 g/l.

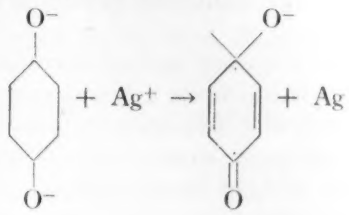
proportion of a compound which reacts somewhat less energetically with sulfites would probably be more effective.

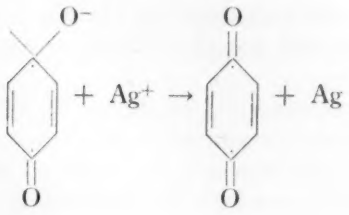
MECHANISM OF THE EFFECT.

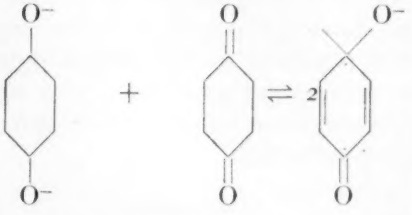
The experimental evidence shows that (*a*) developer in the immediate neighborhood of developing grains is more active than fresh developer; (*b*) this extra activity is very short-lived and is followed by the lowered activity characteristic of exhausted developer; and (*c*) the extra activity is destroyed by sulfite ion.

As mentioned previously, Luther and Frötschner found that developer oxidation products accelerate development. Moreover, the oxidation products are rapidly removed from normal developers by

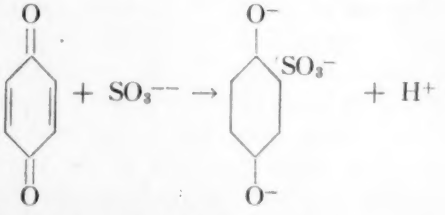
reaction with sulfite. Quinone itself and the semiquinone ion are the most probable oxidation products.[5]







It is probable that the small amount of sulfite present keeps the developer free from these oxidation products until development starts.



Wherever development is taking place, the concentration of quinone and semiquinone rises, activating the developer and accelerating development. The development acts, therefore, as an autocatalytic reaction, and where it once starts it goes rapidly to completion.

This is clearly seen in watching the development of a sensitometric strip of Kodalith film. After a considerable induction period, a very faint image appears. As each step reaches a density of about 0.2, its rate of development increases greatly, owing to the accumulation of

[5] James, T. H., and Weissberger, A., *J. Amer. Chem. Soc.*, 60: 98–104 (1938). Mees, C. E. K., *loc. cit.*, pp. 374–386.

oxidation products, and it then goes completely black in a few seconds. Although development with hydroquinone usually shows an induction period, development after the induction period is normally much less sudden than this.

The accelerated action does not spread very far laterally, since, as the developer diffuses outwards from a developing area, the sulfite in the developer reacts with the oxidation products. The lower the concentration of sulfite, the more slowly will the oxidation products be destroyed, and the farther will the image spread. If the sulfite concentration becomes too low, however, aerial oxidation will maintain a supply of oxidation products throughout the whole developer, resulting in high emulsion speed and fog, but little autocatalytic action and image spreading.

This explanation accounts very satisfactorily for the extremely high contrast and absence of toe produced by Kodalith developers. Unless an area has received enough exposure to produce a density of about 0.2, it does not develop rapidly enough to accumulate oxidation products, and hence is developed in a developer of normal activity. A slight increase of exposure, however, causes it to develop rapidly enough to accumulate oxidation products to the critical point where a powerful autocatalytic action sets in and produces a very high density. A very small increase in exposure, therefore, produces a great change in density: that is, the contrast is very high.

CONCLUSIONS.

The three types of test described: dot spreading, double-layer coatings, and mixed emulsions, all confirm the hypothesis of "infectious development" by formaldehyde-hydroquinone developers. This phenomenon, which is the opposite of the Eberhard effect and Mackie line, depends on the fact that developer in the neighborhood of developing grains is more active than fresh developer, since the oxidation products, which in ordinary developers are rapidly removed by the sulfite, accelerate development. The main function of the formaldehyde appears to be that of a "sulfite buffer," which controls the concentration of free sulfite ions, and hence the rate of removal of oxidation products. Infectious development accounts very simply not only for the filling in of fine clear lines in Kodalith negatives, but also for the extremely high contrast and absence of toe which characterize this type of emulsion and developer.

ROCHESTER 4, NEW YORK,
July 24, 1944.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

ADVANCES IN PLASTICS DURING 1944.

In a review of developments in plastics during 1944, prepared by Gordon M. Kline for publication in *Modern Plastics* and in *Mechanical Engineering*, it is pointed out that the resources and energies of the industry during the past 12 months were fully occupied on wartime assignments. Recently published statistics indicate that the industry has had a tenfold growth during the past decade. The total production of synthetic resins during 1943 was a record figure of 650,000,000 lb. and will probably approach very close to 700,000,000 lb. for 1944.

The year just closed was notable for the steady growth of already familiar products rather than for the introduction of radically new types of compounds. Polyethylene, which has been in commercial production in this country since 1943, is used practically exclusively for the insulation of high-frequency wire and cable. The new high-temperature silicone resin insulation has made it possible to reduce the size and weight of electric motors, to increase greatly the service life of insulation in conventional equipment, and to operate in ambient temperatures and humidities much higher than those permissible for previous types of organic insulation. These resins have provided the basis for the greatest advance in electrical insulation since the advent of glass fiber.

Plastics engineers are well aware that the 400,000,000 lb. plant capacity for the manufacture of styrene, built to meet the requirements of the synthetic rubber program, represents a tremendous potential source of raw material for the production of polystyrene and styrene copolymer resins after the war. The forerunners of numerous developments in this field were announced in 1944.

A high solids water dispersion of vinyl chloride resin, known as Geon latex, has been found suitable for impregnating paper and fabric and for the manufacture of hospital sheeting and foul weather clothing. The latex method eliminates costly solvents and recovery systems, and promotes better adhesion of the resin to fibrous bases.

Resorcinol-formaldehyde resins, which cure rapidly at temperatures from 60° to 150° F. under nearly neutral conditions, have been found to be especially advantageous for assembly bonding of wood and other materials which are deteriorated by the strong acids used in cold-setting urea and phenolic glues.

The terms "sandwich materials" and "expanded (or foamed)

* Communicated by the Director.

plastics" have frequently appeared in the plastics literature of 1944 and are destined to become common parlance during the next few years. The advantages of sandwich construction involving stiff, dense faces separated and stabilized by a thick, light core have long been known. In the aircraft industry considerable interest has been stimulated in sandwich materials by the ever-increasing difficulty of maintaining rigid contours in high-speed aircraft. This type of construction was employed for an experimental resin-glass fuselage built by the Army Air Forces.

The sweeping advances in the development of high-frequency preheating equipment and the utilization of this technique in speeding production and solving heavy-section molding problems make this process outstanding in the roster of 1944 accomplishments in the plastics industry. The record of heatronic molding of industrial parts is filled with examples of amazing reductions in curing time with concomitant savings in manpower and machine hours. Curing time of a propeller block has been reduced from 12 to 2 minutes; telephone handset, 8 to 3 minutes; airplane control pulley, 5 minutes to 1 minute; and an ignition part from 6 minutes to 1 minute.

The adaptation of the injection and extrusion molding processes to thermosetting plastic materials has been another highlight of the year. One machine with motor-driven screw has proved in tests to be satisfactory for injection molding and continuous extrusion of thermosetting and thermoplastic materials and natural and synthetic rubbers. Experimental rocket launching tubes have been made by the continuous extrusion process with this apparatus.

The record of 1944 with respect to utilization of plastics in the war effort has also been primarily one of concentration on items tried and proved in the preceding two years. Bomb tubes, M-52 fuzes, rifle butts, resinous coatings for steel cartridge cases, medicine containers, medical equipment, mess trays, screening, and range-finding equipment, are among the long list of military supplies employing plastics.

The outstanding new application of plastics for military purposes in 1944 was the rocket launching tube. These tubes, which are 10 feet long and have an inside diameter of 4½ inches, are mounted in clusters of three on the underside of fighter planes. Among the advantages attributed to the laminated plastic "flying bazooka," as compared with steel and magnesium tubes, are lighter weight, resistance to corrosion, and reduction of the hazards involved in the event of malfunctioning of the rocket equipment.

The year was marked by a further healthy growth of the technical literature on the properties and testing of plastics; the number of such articles was almost double that of the 1943 review.

Technically, the highlight of the year was the symposium on plastics

held under the auspices of the American Society for Testing Materials in Philadelphia in February. Among the subjects covered were the heat resistance of laminates, effect of environmental conditions on the mechanical properties of plastics, diffusion of water through plastics, stiffness and brittleness of vinyl elastomers, behavior of plastics under repeated stress, testing of high-strength plastics, and creep characteristics of plastics.

The formation of ASTM Committee D-14 on adhesives was announced during the year. Subcommittees on strength properties analytical tests, permanence properties, working qualities, specifications, and nomenclature have been organized. The cooperative efforts of producers, users, and other interested groups in the activities of this committee should lead to uniform testing procedures and valuable data on the properties of all types of adhesives, and promote the development of improved resinous bonding materials.

RELAXATION OF STRESSES IN ANNEALING GLASS.

Because of their high viscosity, glasses and certain so-called plastics are easily undercooled and superheated. As a result, their properties, including viscosity, are usually changing continually while these materials are at temperatures within their annealing ranges, even if the temperature is constant. That is, the changes continue until the materials reach equilibrium conditions at the temperatures at which the annealing takes place. This change in viscosity which accompanies the approach to equilibrium greatly affects the rate at which stresses relax when strained glass is being annealed at a constant temperature.

Since the annealing process for the relaxation of stresses is a very important matter in the production of glass, it has been the subject of many extensive studies, and various attempts have been made to relate the relaxation of the stresses to the time and temperature of annealing. The exponential relation derived from Maxwell's equation for viscous flow, without considering the change in viscosity at a constant annealing temperature, is usually entirely inadequate. Also, the substitute reciprocal relation proposed by Adams and Williamson is generally unsatisfactory.

In RP1637 which will appear in the *Journal of Research* for February, Arthur Q. Tool derives another relation. It is based on Maxwell's equation but takes into account the change in viscosity. To compare its applicability with that of the relation proposed by Adams and Williamson, it has been applied to their data on the relaxation of stresses at various temperatures and in several kinds of glass. This comparison greatly favors the new relation. Fortunately, it also leads to a clearer understanding of the behavior and nature of glass.

SPECTRAL RANGE OF BIOLOGICAL EFFECTIVENESS OF SUNBURN-PREVENTIVE CREAMS.

As noted in *Technical News Bulletin* No. 326 (June 1944), the most efficient sunburn-preventive creams contain substances that are highly opaque to solar radiation of wave lengths 2900Å to 3200Å which are specific in causing sunburn and in producing vitamin D (antirachitic).

Such creams are somewhat transparent to wave lengths 2660Å and shorter; hence they permit the passage of (antirachitic) vitamin D-producing and erythematogenic rays. However, the question has arisen whether ultraviolet solar radiation of wave lengths 2660Å and shorter can reach the earth's surface in sufficient intensity to be biologically effective in producing vitamin-D.

Using a quartz spectrograph, the shortest wave length yet recorded at sea level is 2898Å; and in the high Alps (3.4 kilometers) and in unmanned balloons at a height of 30 kilometers, the shortest wave lengths photographed were slightly longer than 2863Å. Taking into consideration the threshold of blackening of the photographic plate this indicates an intensity of the order of one-billionth of a watt/cm^2.

Using the most sensitive "light counters" (Geiger counters) yet devised and photoelectric cells that respond only to wave lengths of 2680Å and shorter, observers are in disagreement in their observations of incoming radiation (if any) of wave lengths shorter than 2898Å, recorded photographically.

Granting that ultraviolet solar radiation of wave lengths 2680Å and shorter has been detected by means of "light counters," and basing his calculations (1) upon observations of Fabry and Buisson (*Astrophys. J.* **54**, 297, 1921) and that the earth's atmosphere transmits less than 1 part in 2,000,000 of the incident radiation at 2898Å and (2) upon laboratory measurements on ozone (and long columns of air between mountain stations) showing that the transmission of the earth's atmosphere for wave lengths at 2600Å is less than 1/1000 of the value at 2898Å, W. W. Coblentz concludes that the intensity of solar radiation of wave lengths at 2600Å is reduced to less than 1 part in 2,000,000,000 in traversing the earth's atmosphere.

Furthermore, taking into consideration spectral energy measurements, by three sets of observers, in the range of 2900Å to 3100Å, indicating an intensity at 2600Å that is about 1/100 to 1/150 that at 3024Å, outside the earth's atmosphere, Dr. Coblentz deduces that after transmission through the earth's atmosphere the intensity at 2600Å is about one part in 3,000,000,000 of that at 3024Å.

The antirachitic (vitamin D-producing) and the erythematogenic effectiveness of these two wave bands is practically the same. Hence, comparisons are permissible.

In order to insure effective biologic action within a solarium during

the winter, the Council on Physical Medicine of the American Medical Association requires for acceptance that the window glass have a transmission of not less than 30 per cent. at 3024Å, or roughly 3 microwatts/cm² of solar radiation in this wave band. Since sunburn-preventive creams transmit less than 1 per cent. in the wave band at 2600Å, and since the estimated intensity of the incident solar radiation is not greater than a few billionths of the threshold, the conclusion arrived at is that even if ultraviolet solar radiation of wave lengths 2200Å to 2800Å reaches the earth's surface it is biologically ineffective.

This conclusion seems to be supported by the fact that the human skin has not developed a defensive tanning reaction for wave lengths shorter than about 2800Å.

THERMODYNAMICS OF REACTIONS INVOLVED IN PRODUCTION OF HYDROCARBON LIQUID FUELS.

A comprehensive report giving selected "best" values of the thermodynamic properties for a number of reactions fundamental to the production of hydrocarbon liquid fuels from natural gas or coal and hydrogen will appear in the February number of the *Journal of Research*.

This report (RP1634) by Donald D. Wagman, John E. Kilpatrick, William J. Taylor, Kenneth S. Pitzer, and Frederick D. Rossini, was prepared jointly by the thermochemical laboratory and the American Petroleum Institute Research Project 44 at the Bureau, and gives values for the following thermodynamic properties: The heat content function, $(H^\circ - H^\circ_0)/T$, the free energy function, $(F^\circ - H^\circ_0)/T$, the entropy, S°, the heat content, $H^\circ - H^\circ_0$, and the heat capacity, C_p°, for O_2(gas) to 5,000° K, H_2(gas) to 5,000° K, H_2O(gas) to 3,000° K, N_2(gas) to 5,000° K, C(solid, graphite) to 1,500° K, C(solid, diamond) to 1,200° K, CO(gas) to 5,000° K, CO_2(gas) to 3,500° K, and CH_4(gas) to 1,500° K; the standard entropy, S°, for H_2O(liq.) at 25° C; the heat of formation, ΔHf°, the free energy of formation, ΔFf°, and the equilibrium constant of formation, Kf, from the elements, for H_2O(liq.) at 25° C, and for H_2O(gas), CO(gas), CO_2(gas), and CH_4(gas), to 1,500° K; the increment in heat content, ΔH°, and the increment in free energy, ΔF°, for the transition of graphite into diamond, to 1,200° K and to 20,000 atmospheres.

From the foregoing, values were calculated for the increment in heat content, ΔH°, the increment in free energy, ΔF°, and the equilibrium constant, K, for the following reactions:

$$C(\text{solid, graphite}) + CO_2(\text{gas}) = 2CO(\text{gas})$$
$$C(\text{solid, graphite}) + H_2O(\text{gas}) = CO(\text{gas}) + H_2(\text{gas})$$
$$CO(\text{gas}) + \tfrac{1}{2}O_2(\text{gas}) = CO_2(\text{gas})$$
$$CO(\text{gas}) + H_2O(\text{gas}) = CO_2(\text{gas}) + H_2(\text{gas})$$
$$CH_4(\text{gas}) + \tfrac{1}{2}O_2(\text{gas}) = CO(\text{gas}) + 2H_2(\text{gas})$$

$$CH_4(gas) + CO_2(gas) = 2CO(gas) + 2H_2(gas)$$
$$CH_4(gas) + H_2O(gas) = CO(gas) + 3H_2(gas)$$
$$CH_4(gas) + 2H_2O(gas) = CO_2(gas) + 4H_2(gas)$$

A table of the fundamental constants used in the calculations is given.

THE SPECTROGRAPHIC ANALYSIS OF PORTLAND CEMENT.

Spectrographic analysis (quantitative analysis by means of the spectrograph) depends upon the fact that elements can be made to radiate light, the spectrum of which has many lines characteristic of those elements. The relative intensities of lines for different elements bear a complicated but definite relation to the relative quantities, kinds and number of elements in the source. By controlling the excitation used for the source and dividing analytical problems into type cases, empirical data may be obtained which give a quantitative analysis quickly and accurately. Spectrographic methods are ideally suited to problems in which (1) a rapid determination is required, (2) the size of the sample is necessarily small, (3) the concentrations of the elements in question are small, and (4) the chemical method is difficult or uncertain. This subject has been most advanced in the fields of metals and alloys where speed is essential. Its successes there have encouraged its spread to many other analytical problems.

The spectrographic method applied to the determination of the minor elements in portland cement would be expected to result in a great saving in labor and time. This has been found to be particularly true for the alkali elements where the usual chemical methods are difficult, time-consuming and subject to irregularities. Because of the empirical character of the spectrographic method and its dependence upon the nature and state of all the elements present, a special study was required to develop a detailed and reliable procedure for portland cement. In the February *Journal of Research* (RP1633), Armin W. Helz describes a method employing standard mixtures that contain buffer elements (to regulate excitation conditions in the arc), as well as reference elements. A weighed amount of the above material, mixed with the cement, is placed in the crater of a graphite electrode that becomes the lower element of a direct-current arc. The red region of the spectrum is photographed for sodium and potassium and the violet region for lithium. Measurements of the blackening of certain lines of the alkali and the reference elements give the data for the analytical calculations. Analytical curves are required which give the relation between the relative intensities and the concentrations of the elements; they are determined by carrying out exactly the same procedure on series of samples of known compositions.

LIQUID-JUNCTION POTENTIALS AND CALIBRATION OF pH METERS.

The advent of the rugged and stable pH meter of the vacuum-tube type has made comparatively easy the measurement of the pH of most solutions with an accuracy within ± 0.1 pH unit. The manufacture of a large number of medicinals and other products now requires a much greater accuracy in the control of pH, a step which has necessitated the investigation of sources of error formerly considered to be within the uncertainty of the measurements.

In the February issue of the *Journal of Research* (RP1632), George G. Manov, Nicholas J. DeLollis, and S. F. Acree focus attention on one of the largest remaining sources of error (± 0.05 pH unit) in pH measurements, namely, the potential generated at the junction of the standardizing buffer or unknown solution with the saturated potassium chloride solution which forms the electrolyte of the reference electrode. Among the lesser sources of error in pH measurements are the asymmetry potential of the glass electrode, thermal and electrical hysteresis of the calomel electrode, temperature inequalities in the cell system, and unknown salt effects.

In measuring the pH of unknown solutions or in calibrating the scale of a pH meter by the use of two or more buffers, it is the difference between the liquid-junction potentials of the calibrating buffer and of the unknown solution, respectively, in contact with the saturated potassium chloride which must be considered. A method is described by which these differences in liquid junction of two solutions of known pH can be obtained with a precision of ± 0.002 pH unit, and which is independent of the nature of the reference electrode. The experimental data on hydrochloric acid, phthalate, phosphate, phenolsulfonate, borax, and calcium hydroxide buffers containing sodium or potassium chloride and ranging in pH value from 1.081 to 12.38 were also used to check the consistency of the calibration of the scales of pH meters with the various buffer standards recommended by the Bureau for this purpose. The results show that the maximum error caused by the accidental displacement of these buffer standards from their theoretical position in the pH scale by carbon dioxide and other impurities is less than the sensitivity of the average pH meter (0.01 unit). Data are also given for the calibrations of Type 015 and "low-alkali error" glass electrodes of commercial manufacture when tested against seven buffer solutions.

REGION OF USABLE IMAGERY IN AIRPLANE-CAMERA LENSES.

When a lens is used for photographing a distant object, there is a considerable range, measured along the lens axis, wherein a tolerably distinct image can be obtained. The magnitude of this range is termed "depth of focus" and may be computed for an ideal lens with the aid of

geometric optics supplemented by physical optics. In the *Journal of Research* for February (RP1636), Francis E. Washer gives the results of an investigation initiated to ascertain the extent of agreement between theoretical performance of an ideal lens and the observed performance of actual lenses. In the case of several lenses, the entire region of usable imagery was studied, not only for axial imagery but for extra-axial imagery as well, and for various stop openings. The results are summarized in a group of master curves that completely describe the resolving power characteristics of a given lens for both tangential and radial imagery throughout the entire region of usable imagery.

Careful examination of these curves reveals quantitative information on the variation of resolving power and of depth of focus with stop opening and with angular separation from the axis. The manner in which field curvature affects the imagery in a given plane can be clearly seen. The explanation for the betterment of definition in the image plane of a fixed focus camera that accompanies a reduction of stop opening is at once evident from consideration of these master curves. The master curves showing the results of resolving power measurements on several lenses and additional curves showing the degree of agreement between theoretical and observed performance are given in the report.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, FEBRUARY 21, 1945.

The regular Monthly Meeting of The Franklin Institute was called to order by Mr. Charles S. Redding, President, after the National Anthem had been played.

Upon motion, duly seconded and carried, the minutes of the December Meeting as printed in the January JOURNAL were approved.

The Chairman called upon the Secretary of the Institute, Dr. Henry Butler Allen, who made the following report on Membership:

Active	48
Associate	19
Student	32
Total	99

Total Membership as of January 31, 1945 5,454

Dr. Allen then stated that the Board of Managers of The Franklin Institute proposed the name of the Franklin Medalist for 1945 for Honorary Membership in The Franklin Institute—

Dr. Harlow Shapley, Director,
Harvard College Observatory.

Upon motion duly seconded and carried, Dr. Shapley was so elected.

The Chairman then introduced the speaker of the evening, Dr. Frank B. Jewett, President of the National Academy of Sciences and Member of the National Defense Research Committee, who spoke on "War Work of the National Academy of Sciences and the National Research Council." He gave an interesting account of the present work of the National Academy of Sciences and the National Research Council and the work of establishing the Research Board for National Security which the Secretaries of War and Navy have asked the Academy to set up.

The Meeting was dismissed with a rising vote of thanks to the Lecturer.

HENRY BUTLER ALLEN,
Secretary.

February 21, 1945.

COMMITTEE ON SCIENCE AND THE ARTS.

(Abstract of Proceedings of Stated Meeting held Wednesday, February 14, 1945.)

HALL OF THE COMMITTEE,
PHILADELPHIA, FEBRUARY 14, 1945.

MR. JAMES D. LEE, JR., *in the Chair.*

The following reports were presented for final action:

No. 3140: Work of Edwin A. Link in Flight Training.

This report recommended the award of a Howard N. Potts Medal to EDWIN A. LINK, of Binghamton, New York, "For valuable contributions in the field of training devices for aviators."

No. 3145: Brown Medal.

This report recommended the award of a Frank P. Brown Medal to GILMORE DAVID CLARKE, of New York City and Ithaca, New York, "In consideration of his technical ability, foresight, initiative and outstanding leadership in the field of town and city planning."

No. 3146: Clamer Medal.

This report recommended the award of a Francis J. Clamer Medal to ZAY JEFFRIES, of Cleveland, Ohio, "For his meritorious contributions to the science of metals, which he has placed on a new and more intelligible basis."

JOHN FRAZER,
Secretary to Committee.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from nine o'clock A.M. until five o'clock P.M., Wednesdays and Thursdays from two until ten o'clock P.M.

RECENT ADDITIONS.

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NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Two Constants Connected with the Theory of Prime Numbers.—H. M. TERRILL AND LUCILE SWEENY. The sum of the reciprocals of the prime numbers may be written

$$1/2 + 1/3 + 1/5 + 1/7 + 1/11 + \cdots + 1/p \rightarrow \log \log p + B, \quad (1)$$

where B is a constant $.261\cdots$ (Ingham, "The Distribution of Prime Numbers," Cambridge, 1932, p. 22, Eq. 23).

The value of the constant B is obtained indirectly by means of a second constant D, from the relation

$$B = C + D, \quad (2)$$

where C is Euler's constant, $.5772\cdots$.

It is the object of this article to determine the value of the two constants, B and D, each to 23 places. Equation (2) is derived by Ingham (*op. cit.*, p. 23) in the form

$$B = C + \sum_p [\log (1 - 1/p) + 1/p], \quad (3)$$

where the summation is to be taken over all prime numbers. Thus the constant D is given by

$$D = \sum_p [\log (1 - 1/p) + 1/p] \quad (4)$$

and the value may be calculated directly. The series in (4) is rather slowly convergent, but it is possible to shorten the work by taking advantage of certain numbers already tabulated.

If we expand the quantity $\log (1 - 1/p)$ by the logarithmic series for each value of p, (4) becomes

$$D = \sum_p (-1/2p^2 - 1/3p^3 - 1/4p^4 - \cdots - 1/np^n - \cdots), \quad (5)$$

where as before the summation is over all prime numbers. This expression can be rearranged by collecting the terms with the same exponent, thus:

$$D = \sum_{n=2}^{\infty} (-1/2^n - 1/3^n - 1/5^n - \cdots - 1/p^n - \cdots) \quad (6)$$

which may be written

$$D = -1/2\ \Sigma_2 - 1/3\ \Sigma_3 - 1/4\ \Sigma_4 - \cdots - 1/n\ \Sigma_n - \cdots \quad (7)$$

where $\Sigma_n = \sum_p 1/p_n$.

The $\sum_n$ are tabulated in the extensive tables of H. T. Davis ("Tables of the Higher Mathematical Functions," Bloomington, Indiana, 1935, Vol. 2, p. 249) where the values for n odd have been supplied by Davis, and the others were recomputed to check existing values. Taking 73 terms of the series in (7) we find

$$D = -.31571\ 84520\ 53890\ 07685\ 108\cdots.$$

The value of this constant was first computed by Merrifield (*Proceedings of the Royal Society*, **33**: 7, 1881).

Merrifield gave the value of D to 15 places. When checking this with our value, we find one digit of his, the 11th, not in agreement with ours. Merrifield gives 7 for the eleventh digit while we find 5. It was noted by Glaisher (*Quarterly Journal of Mathematics*, **25**: 347, 1891) that some of Merrifield's statements in regard to this constant are not entirely correct, but Glaisher's criticism does not apply to the method of computation. We have accordingly computed the value of D by Merrifield's method to check the value obtained from (7).

The series used by Merrifield, which is based on a theorem due to Moebius, is given by the following equation:

$$D = -1/2 \log S_2 - 1/3 \log S_3 - \cdots \pm 1/n \log S_n \cdots, \qquad (8)$$

where

$$S_n = \sum_1^\infty 1/x^n,$$

and n takes only values that do not contain a square factor, the sign of the term being + or − according as the number of factors of n is even or odd. The values of $\log S_n$ are also tabulated by Davis (*op. cit.*). Taking 46 terms of this series, we find the value of D as before, the two methods showing agreement to the 23rd place. The value of D being substituted in (2) we obtain the value for B

$$B = .26149\ 72128\ 47642\ 78375\ 543\cdots.$$

We wish to thank Prof. R. C. Archibald, editor of *Mathematical Tables and Aids to Computation*, for information regarding the references to the work of Davis and Merrifield.

BOOK REVIEW.

WHAT ARE COSMIC RAYS? by Pierre Auger. Translated from the French by Maurice M. Shapiro. 127 pages, 22 plates, diagrams, 14 × 20 cms. Chicago, University of Chicago Press, 1945. Price $2.00.

The book is written primarily for the reader who lacks a technical knowledge of physics and includes also the physicist who wants to get a quick view of what has been done in this new field. It, therefore, is written in an easy-to-read style, first providing a background through a description of the ionization phenomena in matter, the progress of the study of the "rays from the sky," and the definite determination of the cosmic rays. The story assumes the form of a mystery "thriller" in which scientists play the part of super detectives, yet the serious, factual side is always foremost. Mysteries are unraveled, followed closely by proven explanations. Various problems that have become evident are given and the means of their solution, and the many unsolved problems are pointed out. "Question No. 1," so called, makes itself evident through most of the book. It is:—What is the primal origin of the cosmic rays? A survey of the principal hypotheses which have been advanced rejects the theories which see within the solar system the possible source, rejects the stars, rejects regions of space where matter exists only in very low concentration, and finally rejects the action of magnetic fields such as those which may exist in the neighborhood of double stars.

The book reveals that we know a great deal about cosmic rays. What is more important, it reveals some of the things that we don't know, which offer a challenge and a stimulation—a healthy condition for the pushing back of the unknown. It is an up-to-date evaluation of the state of knowledge of this fascinating subject.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

The Physics of Flight, by Alfred Lande. 122 pages, illustrations, 15 × 23 cms. New York, Reinhold Publishing Corporation, 1945. Price $2.50.

What are Cosmic Rays? by Pierre Auger. Translated from the French by Maurice M. Shapiro. 128 pages, XXII plates, diagrams, 14 × 20 cms. Chicago, University of Chicago Press, 1945. Price $2.00.

Soul of Amber, The Background of Electrical Science, by Alfred Still. 274 pages, 14 × 22 cms. New York-Toronto, Murray Hill Books Inc., 1944. Price $2.50.

An Introduction to Electronics, by Ralph G. Hudson. 97 pages, diagrams and illustrations, 15 × 22 cms. New York, The Macmillan Company, 1945. Price $3.00.

CURRENT TOPICS.

Future for Soybeans.—Greatly expanded use of soybeans in the United States both in industry and for food and livestock feed were forecast by Dr. E. O. May, Chief of the Bureau of Agriculture and Industrial Chemistry in the U. S. Department of Agriculture and in charge of the Agricultural Research Administration's four Regional Research Laboratories.

"It is possible to mention only a few of the many lines of research now being prosecuted," Dr. May told the twenty-fifth annual meeting of the American Soybean Association at Urbana, Ill., in reviewing the future possibilities of soybean protein in industry. "In the past, plastic materials, adhesives, and paper coatings have resulted from such research, and films and fibers made from soybean protein now seem destined to assume industrial importance.

"The search for new uses is being constantly furthered by advances in our knowledge of the chemical and physical properties of soybean protein. It is a most complicated substance, but those features which contribute to its complexity are the very ones which lend it valuable properties. The intricacies of the various protein molecules are slowly but certainly being unravelled, and with each new advance in our understanding of the chemistry of soybean protein, new possibilities are presented for the improvement of products made from soybean meal with consequent enhancement of their value."

In industrial fields Dr. May also forsees a vast future for soybean oil, particularly in the production of coatings and other polymeric products. Soybean oil, he described as a mixture of materials, some of which are ordinarily considered best suited for use as foods and others that are best used in paints and varnishes. "In the past," he said, "it has not been economically practical to separate these constituents, but means for doing so have recently been developed. The perfection and intelligent application of fractionation processes, separating individual components for specific uses, is undoubtedly the key to future expansion in the industrial use of soybean oil."

In their simplest application, according to Dr. May, the new fractionating tools will be used to separate soybean oil into two fractions, one of which has been found to be an exceptionally fine edible oil, and the other of which is excellent for use in paints and varnishes. Both portions will be worth much more than the original oil, and exploitation of the process on a large scale would alleviate to some extent possible surpluses of edible oils in the post-war future.

Concerning this process Dr. May further stated:

"Fractionation of soybean oil has far-reaching possibilities since the principal constituent is linoleic acid, which comprises more than 50 percent of the various fatty acids making up the oil molecule. This acid appears to be the most valuable product obtainable from the ordinary fats and oils, from the standpoint of industrial utilization. Linoleic acid can be used as the initial raw material for making a wide variety of synthetic products. Typical

examples, developed at the Northern Regional Research Laboratory, are Norepol, a rubber substitute, and Norelac, a new resin having remarkable properties. These products represent only the beginning of what will undoubtedly in due time become a very large industry based on the commercial separation of the constituents of soybean oil and their use as industrial raw materials."

Dr. May pointed out also that soybeans contain many minor constituents of industrial importance, a few of which are now being recovered. He cited as examples the phosphatides, sterols, vitamins, antioxidants, saponins. In most cases, he said, the quantities of these products potentially available in soybean oil are sufficient to warrant extensive research on possible uses in industry, and significant developments are to be expected.

Although emphasizing the possibilities of the further industrial uses of soybeans Dr. May said that it should not be assumed that industrial utilization approaches in importance the requirements for soybeans in the food and feed industries.

R. H. O.

ARMY AND NAVY NOTES.

The Jet Plane.—General Electric engineers can now disclose some of the advantages of the jet-propelled planes which the Company is equipping with gas turbine engines, among them the fact that the propellerless craft can take-off almost from scratch, without the warm-up required by conventional ships.

"Full thrust from the gas turbines is available in approximately 30 seconds after they are started, thereby eliminating any delay for warming up the engines," according to Company engineers.

Disclosure of this and other information about the jet plane power plant was permitted by the War Department concurrent with announcement of the news that Allied jet-propelled planes have been used successfully by the British against the German robot bombs. General Electric is building the gas turbines for the American version of the revolutionary-type aircraft, described as a single-seater, high-altitude fighter.

No Visible Exhaust. "The jet plane turbines operate satisfactorily with either kerosene or gasoline, the former fuel having less tendency than gasoline to ignite in the open air," G-E engineers state. "When the jet plane is in full flight, no visible flames come out of the exhaust. And because continuous ignition is not required, radio interference and high-altitude ignition problems are eliminated.

"The turbine is a compact, self-contained unit which has only one moving part. Air is taken into the turbine, compressed, and passed into chambers where its temperature is increased by the combustion of fuel, creating hot gases which are discharged through a tailpipe nozzle.

"Forward thrust of the plane is developed as the reaction to this high-velocity discharge in accordance with the physical law that for every action there is an equal and opposite reaction. A common example of this principle is the rotary lawn sprinkler, which is rotated in one direction by jets of water directed in the opposite direction."

Little Vibration. "Because this thrust power is produced directly without requiring gearing or a propeller, vibration is minimized, and reduced weight

per horsepower is made possible," the G-E engineers explain. "Thrust produced by the turbine is easily regulated and overhauls may be accomplished quickly when necessary."

The gas turbines for the American type jet planes were developed by General Electric on modifications of a British design conceived by Group Captain Frank Whittle, of the Royal Air Force. Six months after the project was assigned to G.E. by the Army Air Forces in 1941, the Company successfully modified, developed, and produced the first jet-propulsion gas turbines in America.

They powered the P59A built by Bell Aircraft which made the first successful jet plane flight in this country in October, 1942. Since then the jet-propelled plane has been undergoing great development. Recently, Gen. H. H. Arnold pointed out that great hopes are being held for it for future commercial as well as military use.

Although production schedules for jet-propelled planes have not been revealed, General Electric was permitted to announce last July that it was converting its second largest wartime plant of more than 600,000 square feet of floor space for the exclusive production of jet propulsion aircraft turbines.

R. H. O.

Foxhole Smoke Generator.—A portable mechanical smoke generator for use on jungle trails, mountain passes and beachheads is the newest smoke-screening device developed by the Chemical Warfare Service. Small enough to be carried in the back seat of a jeep or other similar craft, when in use it can be concealed behind a bush or in a dugout. With favorable wind conditions this midget fog machine can blot out an area five miles long and about 200 yards wide.

Known as the M-2 smoke generator, or the Besler, the device is no larger than a soldier's foot locker and weighs only 180 pounds when empty. It is about 1/20th as bulky as the army's large M-1 smoke generator, the principle of which was developed in General Electric's research laboratory by Dr. Irving Langmuir and Vincent J. Schaefer. The M-1 machine, truck or trailer driven, protected allied troops from aerial attacks during the invasions of North Africa, Sicily and Italy.

The foxhole fog generator will not make the M-1 obsolete. While the small generator moves into the front lines, producing half as much smoke as its big brother, the larger machine will continue to send clouds of milk-white smoke around airfields, seaports, cities and permanent large installations.

The new generator, 34 inches long, 23 inches high and 20 inches wide, when equipped with sliding handles, can also be carried in stretcher fashion by two men. The device consists of a miniature metal boiler, an air blower and a small gear pump, all operated by a one-cylinder gasoline motor made of aluminum. It can make smoke in 40 seconds from a cold start.

The pump forces a mixture of fog oil and water through a series of spiral coils inside the boiler, where a small gasoline burner heats the liquid. The oil-and-water mixture vaporizes inside the heating coils and emerges from four small vents on a T-shaped outlet valve at the front of the boiler, then condensing into a snow-white steam. The blower is used to drive air for combustion into the torch-like burner.

The hollow base of the machine holds tanks for enough water and gasoline for more than an hour of operation. The fog oil is drawn into the generator from oil drums or five-gallon "blitz" cans by means of a short rubber hose.

Consuming 50 gallons of fog oil per hour, the foxhole generator also uses about five gallons of gasoline and five gallons of water during that period. It is completely automatic in operation, and there are no gages to match.

The smoke is not harmful and is of no discomfort to troops.

R. H. O.

Portable Flashing Beacon for Temporary Allied Airports.—A portable flashing beacon, using a gaseous-discharge lamp of a type originally developed for ultra high-speed photography, has been designed by General Electric Lighting Division engineers for identification signalling at temporary Allied airports. It uses only one-third the power required for a stationary airport rotating beacon and weighs only one-third as much. Mounted on a collapsible mast, it produces a coded flash of such intense, split-second brilliance that its exact candlepower cannot be measured.

Power for this flashing beacon is supplied by a portable gas-engine generating set. The beam unit with its associated control unit fits into a small steel trunk with compartments for storing the beacon head, cable, lamps and tubes. Equipment includes a transformer, vacuum tube rectifier, capacitor, flasher, and an electronic relay for controlling the signal flashes. A choice of five codes is possible simply by rotating a selector switch built into the control unit.

The flashes are obtained by charging the capacitor by means of the transformer and rectifier and discharging it through the lamp. Cam-operated contacts driven by a synchronous motor actuate the electronic relay to time the frequency of the flashes. The intensity and duration of the flashes is inherent in the design of the lamp and power pack. The coding consists of timing the interval between flashes.

The beacon lamp is enclosed, for protection and light control, in a dome type Fresnel lens which emits the light at all angles above horizontal with maximum candlepower at about five degrees above horizontal. The lamp operates at 2000 volts d-c potential. The complete unit operates on 115 volts a-c.

As a safety measure the outfit is provided with an electrical interlock so that when the auxiliary cover over the power pack is lifted the main power supply is shut off and the high voltage condenser short-circuited through a safety discharge relay. This relay operates whenever the power supply is shut off.

R. H. O.